

SPECIFIC NET WORK AND MEAN EFFECTIVE PRESSURE BASED THERMODYNAMIC ANALYSIS AND OPTIMIZATION OF IDEAL ATKINSON CYCLE

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ABSTRACT: In this study, the criteria for maximum values of specific net work and mean effective pressure in the air standard Atkinson cycle, which is the ideal thermodynamic cycle of internal combustion engines used in hybrid electric vehicles, were examined. For this, it was assumed that the cycle operates in a certain temperature range. Thus, while the maximum temperature of the cycle is constant, the compression ratio was optimized for maximum specific net work and maximum mean effective pressure. A case study was conducted for this study. For the case study, assuming the geometric expansion ratio of 12, the maximum temperature value of the cycle was determined as 1961.923 K. Based on this maximum temperature value, the maximum values of the specific net work and the ambient effective pressure were determined as 580.139 kJ/kg and 373.857 kPa, respectively. In addition, the geometric compression ratios for the maximum values of the specific net work and the ambient effective pressure were determined as 15.462 and 31.063, respectively. Looking at the geometric compression ratio values of today's engines, it was seen that these values were closer to the compression ratio at which the maximum specific net work was achieved. It was observed that the thermal efficiency increased when the compression ratio was optimized for the conditions where the maximum average effective pressure was achieved. The results obtained from this study are particularly attractive to engine designers.

Keywords: Atkinson Cycle, Optimal Compression Ratio, Mean Effective Pressure, Specific Net Work

İdeal Atkinson Çevriminin Özgül Net İş ve Ortalama Efektif Basınç Temelli Termodinamik Analizi ve Optimizasyonu

ÖZ: Bu çalışmada, hibrit elektrikli taşıtlarda kullanılan içten yanmalı motorların ideal termodinamik çevrimi olan hava standart Atkinson çevriminde özgül net işi ve ortalama efektif basıncının maksimum değerleri için kriterler incelenmiştir. Bunun için çevrimin belirli sıcaklık aralığında çalıştığı varsayılmıştır. Böylelikle çevrimin maksimum sıcaklığı sabit iken maksimum özgül net iş ve maksimum ortalama efektif basınç için sıkıştırma oranı optimize edilmiştir. Çalışma için bir durum çalışması yapılmıştır. Vaka çalışması için geometrik sıkıştırma oranının 12 olduğu varsayılarak çevrimin maksimum sıcaklık değeri 1961.923 K olarak belirlenmiştir. Bu maksimum sıcaklık değeri referans alınarak özgül net işin ve ortamala efektif basıncın maksimum değerleri sırasıyla 580.139 kJ/kg ve 373.857 kPa olarak belirlenmiştir. Ayrıca özgül net işin ve ortamala efektif basıncın maksimum değerleri için geometrik sıkıştırma oranıları ise sırasıyla 15.462 ve 31.063 olarak belirlenmiştir. Günümüz motorlarının geometrik sıkıştırma oranı daha yakın olduğu görülmüştür. Maksimum ortalama efektif basıncın elde edildiği sıkıştırma oranına daha

oranı optimize edildiğinde termal verimin de arttığı gözlemlenmiştir. Yapılan bu çalışmadan elde edilen bulguların özellikle motor tasarımcılarının dikkatini çekecek niteliktedir.

Anahtar Kelimeler: Atkinson Çevrimi, Optimum Sıkıştırma Oranı, Ortalama Efektif Basınç, Özgül Net Iş

1. INTRODUCTION

Internal combustion engines are today primary energy conversion machines for both mobile systems and stationary systems (Grohe, 2003). The main reason for the widespread use of internal combustion engines is undoubtedly the ease of supply and storage of energy. However, when evaluated in terms of energy conversion efficiencies, internal combustion engines have dramatically lower energy conversion efficiencies compared to electric motors.

Today, internal combustion engines working with gasoline or diesel fuel are widely used (Halderman and Mitchell, 2014). In classical thermodynamics, internal combustion engines are idealized as air standard closed cycles. Internal combustion engines running on gasoline fuel are known as spark ignition engines and are idealized as the Otto cycle in classical thermodynamics. In the ideal Otto cycle, both the heat input and heat output processes take place under constant volume conditions. Internal combustion engines running on diesel fuel are known as compression ignition engines and are idealized as the Diesel cycle in classical thermodynamics (Ferguson and Kirkpatrick, 2015). In the ideal diesel cycle, the heat input process takes place under the constant pressure condition, and the heat output process takes place under the constant volume condition (Rajput, 2009-1). Today, however, there are also engines operating according to the principle of different thermodynamic cycles. The most well-known of these is the Atkinson cycle engine used in hybrid-electric vehicles. In the ideal Atkinson cycle, the heat input process takes place under the constant volume condition as in the Otto cycle, and the heat output process takes place under the constant pressure condition. The reason why Atkinson cycle engines are preferred in hybrid-electric vehicles is their low power density (maximum power per unit engine volume) and high thermal efficiency compared to the Otto cycle engine (Boggs et al., 1995). In other words, Atkinson cycle engines can operate in more economic conditions than Otto cycle engines.

In the Atkinson cycle, the power stroke is longer than the compression stroke because the heat output process takes place at constant pressure. The fact that the power stroke was longer than the compression stroke in the early version of the Atkinson-cycle engine, invented in 1882, was realized thanks to the complex mechanism proposals. Today, instead of these complex mechanisms, modern Atkinson cycle engines can be obtained more practically with valve timing regulation (Zhao et al. 2012). Thermodynamic analysis of heat engines is included in the thermodynamics textbooks under the heading "Gas Power Cycles". When a few of these thermodynamics textbooks were examined, it was seen that the Otto, Diesel and Dual cycles, which are the ideal cycles of piston engines, and the Brayton cycle, which was the cycle of gas turbine engines, are mentioned (Borgnakke and Sonntag, 2020; Ganesan, 2018; Whitman, 2020; Cengel et al., 2011; Pauken, 2011; Potter and Somerton, 2014; Rajput, 2009-2; Schmidt, 2019; Balmer, 2011; Wu, 2007; Nag, 2013; Moran et al., 2010). In these books, it was seen that the effect of compression and expansion ratio on thermal efficiency for ideal cycles of reciprocating engines is examined and these cycles are compared. However, the thermal efficiency expressions are presented only depending on the compression and expansion ratio. But, in the ideal Carnot cycle with maximum thermal efficiency, the expression of thermal efficiency is expressed depending on the temperatures of the cold and hot heat source. For this reason, when examining the ideal cycles of reciprocating engines, it may be more appropriate to make an analysis by analogy with the Carnot cycle for the case where the heat source temperatures are constant. However, the cycle analyzes in the thermodynamics textbooks are made for constant heat input. Thus, although it is not very realistic, the thermal efficiency converges to 100% as the compression ratio and expansion ratio increase for constant heat input. In response to this rising thermal efficiency, the maximum temperature and pressure of the cycle increase immeasurably. However, as the compression ratio and expansion ratio increase, the specific net work and mean effective pressure values become concepts that increase with the same characteristic as the thermal efficiency. In other words, since the specific net work and mean effective pressure values have the same tendency as the thermal efficiency, it is observed that these values increase with the compression ratio and expansion ratio. If the maximum temperature is kept constant instead of the heat input in the cycle, the maximum values for specific net work and mean effective pressure are also limited (Figure 1).

For any gas power cycle, as long as the compression ratio increases, the theoretical maximum thermal efficiency converges to the thermal efficiency of the Carnot cycle operating within the same temperature limits. For extremely high compression ratio the thermal efficiency is extremely high and its limit is the Carnot cycle efficiency. However, under these conditions, the specific net work of the cycle converges to zero. The mean effective pressure, which is related to the power density (maximum power per unit engine volume) of the engine, is also very low at a very high compression ratio. This shows that considering only thermal efficiency in the evaluation of gas power cycles does not really mean anything. For this reason, it is seen that it would be more accurate to consider maximum specific net work or maximum mean effective pressure values instead of high thermal efficiency conditions for gas power cycles.

Researchers have studied systems consisting of reversible, endoreversible or irreversible models for gas power cycles. Although it has been assumed that the cycles work according to the idealizations in classical thermodynamics in many studies, it was also seen that quasi-realistic assumptions are applied, such as the cycles occur in a finite time interval and the specific heats change with temperature (Palaci and Gonca, 2020; Ebrahimi, 2021; Chen et al., 2007; Ding et al., 2011). Palaci and Gonca (2020), made a performance evaluation by limiting the maximum cylinder temperature for a diesel engine cycle (dual cycle) according to the melting degrees of various materials. Of course, as expected, performance improves as the maximum temperature increases. In this case, it is concluded that the engine must reach high temperature in order to improve its performance. This also applies to the improvement of the Carnot cycle efficiency. Ebrahimi (2021), presented a method for the comparison of engine cycles under the condition of maximum thermal efficiency in his study. Here, the temperature range of 2600-3000 K, which is the metallurgical temperature limit, is considered as the maximum temperature and a cycle is compared according to the maximum thermal efficiency. In this study, it was concluded that increasing the maximum temperature increases the thermal efficiency and the concepts of specific net work and mean effective pressure are ignored. Chen et al. (2007), made optimizations based on power, thermal efficiency and entropy generation rate for irreversible universal heat engines in their study. In this study, power cycles operating between two heat sources are investigated by considering irreversibility. Thus, the performances of the cycles were compared on the basis of thermal efficiency. Ding et al. (2011) presented an optimization for reversible cycles in terms of economy and efficiency in their study. In this study, the thermal efficiency parameter has been the focus. It is very interesting that in these studies (Palaci and Gonca, 2020; Ebrahimi, 2021; Chen et al., 2007; Ding et al., 2011) a systematic analysis of a model that maximizes specific cycle net work and mean effective pressure values was not envisaged. Gonca and Şahin (2022), a new high performance engine cycle consisting of five processes and a constant temperature process was developed using new numerical methods and compared with the classical double cycle. Şahin and Gonca (2021) performed thermodynamic analyzes for a reversible dual cycle in their study. Wang et al. (2021) derived the expressions of power output and thermal efficiency for the model by creating an irreversible continuous flow Lenoir cycle model using the theory of finite time thermodynamics. Kim et al. (2022), in this study, ideal cycles with finite heat capacity ratios are theoretically investigated to maximize power generation using an sequential Carnot cycle model. Costea et al. (2021) modeled the irreversible Carnot cycle motor operating as a closed system using the direct method for finite speed and the first law of thermodynamics. When these studies are examined in detail, thermal efficiency is focused on as a performance criterion.

Gas power cycles are expected to output work or power through cycles, and it is desirable that this output work or power be as much as possible from a unit fuel or unit engine displacement point of view. These performance parameters are directly related to the concepts of specific net work or mean effective pressure. Also, boundary conditions for the maximum value of specific net work and mean effective pressure are rarely discussed. However, it is interesting that in all thermodynamics textbooks the topic of gas power cycles is explained with a focus on high thermal efficiency. Although true for internal combustion engines, this unrealistic approach is misleading, with the sole aim of increasing thermal efficiency. As it is known, it is desirable for hybrid electric vehicles to be more economical than conventional vehicles. However, it is known that the thermal efficiency of the ideal Atkinson cycle is higher than the ideal Otto cycle under the same conditions. However, it is unrealistic to evaluate the Atkinson cycle solely on the thermal efficiency focus. Therefore, as a different approach, it is more accurate to determine the optimal value of the compression (or expansion) ratio for the maximum specific net work or maximum mean effective pressure of the Atkinson cycle. In this study, the compression (or expansion) ratio is optimized for both maximum specific net work and maximum mean effective pressure for the ideal Atkinson cycle. In the study, firstly, the classical thermodynamic solution was presented and then the equations for maximum specific net work and maximum mean effective pressure are obtained. Here, the ideal Atkinson cycle operating in the same two constant temperature ranges is used for comparison. In this way, it was possible to evaluate the Atkinson cycle as the equivalent of the Carnot cycle. Thus, the compression ratio was optimized for maximum specific net work and maximum average effective pressure while the maximum temperature of the cycle is constant.

2. THEORETICAL MODEL

In Figure 1, the Temperature-Entropy and Pressure-Volume diagrams of the ideal Atkinson cycle are presented. In studies where parametric comparisons are made, idealizing the cycle as much as possible is an accepted method in the literature to simplify the calculations. Therefore, this method was also applied in this study. Constant specific heat was assumed for the theoretical model. It was also assumed that the working fluid is ideal air. Compression and expansion processes were assumed to occur isentropically.



Figure 1. Ideal Atkinson cycle

In Figure 1, an Atkinson cycle operating in the T_L (sink temperature) and T_H (reservoir temperature) temperature range was created. While T_L value was accepted as ambient temperature, T_H is determined as a value slightly higher than T_{max} value, which is the maximum temperature of the cycle. In this section, first of all, classical thermodynamic expressions for the ideal Atkinson cycle were presented, and then the necessary equations for optimization were created.

2.1. Ideal Atkinson cycle

 q_1 and q_2 are specific heat transfers for input and output, respectively:

$$q_{1} = \int_{T_{2}}^{T_{3}} du = \int_{T_{2}}^{T_{3}} c_{\nu} dT = c_{\nu}(T_{3} - T_{2}) > 0$$

$$(1)$$

$$q_2 = \int_{T_4} dh = \int_{T_4} c_p \, dT = c_p (T_1 - T_4) < 0 \tag{2}$$

Accordingly, w_{net} , which is the specific net work of the cycle, can be expressed as:

$$w_{\rm net} = q_1 - |q_2| = c_{\nu}(T_3 - T_2) - c_p(T_4 - T_1)$$
(3)

The following entropy equation is used for the processes taking place in the cycle:

$$\int ds = \int c_v \frac{dT}{T} + \int R \frac{dv}{v} = \int c_p \frac{dT}{T} + \int R \frac{dp}{p}$$
(4)

To simplify the formulation, $v_1/v_2 = \varepsilon_1$ and $v_4/v_3 = \varepsilon_2$ relationships are created between the volumes. Here, ε_1 and ε_2 are compression ratio and expansion ratio, respectively. While various mechanisms are used to obtain these ratios in classical Atkinson-cycle engines, these ratios are obtained by valve timing in today's modern Atkinson-cycle engines. For this reason, while ε_2 is the geometric compression ratio, ε_1 is also expressed as the compression ratio obtained due to valve timing. As it will be remembered, while $\varepsilon_1 = \varepsilon_2$ in the ideal Otto cycle, it becomes $\varepsilon_1 < \varepsilon_2$ in the ideal Atkinson cycle. According to Equation 4, the entropy change in the heat input and heat output processes in the cycle is expressed $s_3 - s_2 = c_v \ln(T_3/T_2) > 0$ and $s_1 - s_4 = c_p \ln(T_1/T_4) < 0$.

Accordingly, the mean temperatures T_{m1} and T_{m2} are expressed as follows:

$$T_{\rm m1} = \frac{q_1}{s_3 - s_2} \tag{5}$$

$$T_{\rm m2} = \frac{q_2}{s_1 - s_4} \tag{6}$$

The mean temperature expressions here describe the ideal Carnot cycle equivalent temperature range of the ideal Atkinson cycle. In other words, this Atkinson cycle can be accepted as an ideal Carnot cycle operating in the temperature range of T_{m1} and T_{m1} . As a result, as the numerical difference between T_{m1} and T_{m1} increases, the thermal efficiency also increases.

The thermal efficiency expression for the ideal Atkinson cycle is as follows:

$$\eta_{\rm th} = \frac{w_{\rm net}}{q_1} = 1 - \frac{k}{\varepsilon_1^{k-1}} \frac{\frac{\varepsilon_2}{\varepsilon_1} - 1}{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^k - 1} = 1 - \frac{T_{\rm m2}}{T_{\rm m1}} \tag{7}$$

As seen here, the thermal efficiency of the ideal Atkinson cycle, η_{th} is related to ε_1 and ε_2 . However, the $\varepsilon_2/\varepsilon_1$ value is related to the T_3/T_2 value. In addition, the T_2 value is only related to ε_1 . In this case, it is the ε_1 or ε_2 value that is the main determinant. Because ε_2 and ε_1 are related to each other.

The mean effective pressure can be defined as:

$$p_{\rm me} = \frac{w_{\rm net}}{v_4 - v_2} \tag{8}$$

The following equations are defined for the exergy destruction occurring in the heat input and heat output processes in the ideal Atkinson cycle:

$$x_{\text{dest},1} = T_0 \left[(s_3 - s_2) - \frac{q_1}{T_H} \right]$$
(9)

$$x_{\text{dest},2} = T_0 \left[(s_1 - s_4) + \frac{q_2}{T_L} \right]$$
(10)

$$x_{\rm dest} = x_{\rm dest,1} + x_{\rm dest,2} \tag{11}$$

The expended exergy for a heat engine is the reduction in the exergy of heat transferred to the engine, which is the difference between the exergy of heat supplied and the exergy of heat dissipated. The following equation is used for the expended exergy:

$$x_{\rm exp} = \left(1 - \frac{T_0}{T_H}\right) q_1 \tag{12}$$

Accordingly, the thermodynamic efficiency can be expressed as:

$$\eta_{II} = 1 - \frac{x_{\text{dest}}}{x_{\text{exp}}} \tag{13}$$

2.2. Optimization of ε_2 for maximum w_{net}

Assuming an ideal Atkinson cycle operating in the range T_1 to T_3 , shown in Figure 1, the relationship between these two temperatures can be defined as $T_3/T_1 = \varepsilon_2^k/\varepsilon_1 = \tau$. Here, if the independent variable is ε_2 , the first and second derivatives of w_{net} with respect to T_4 are as follows:

$$\frac{dw_{\rm net}}{dT_4} = c_p \left(\frac{T_3 T_1^k}{T_4^{k+1}} - 1 \right) \tag{14}$$

$$\frac{d^2 w_{\text{net}}}{dT_4^2} = -c_p (k+1) \frac{T_3 T_1^k}{T_4^{k+2}} < 0 \tag{15}$$

Here, the first derivative expression is set to 0 and the optimal T_4 expression, which maximizes the w_{net} , is obtained as follows:

$$T_{4,\text{opt}} = (T_3 T_1^k)^{\frac{1}{k+1}} = T_1 \tau^{\frac{1}{k+1}}$$
(16)

Accordingly, optimal ε_2 and optimal ε_1 expressions for maximum w_{net} are obtained as follows:

$$\varepsilon_{2,\text{opt}} = \tau^{k/(k^2 - 1)} \tag{17}$$

$$\varepsilon_{1,\text{opt}} = \varepsilon_{2,\text{opt}}^{1/k} = \tau^{\frac{1}{k^2 - 1}} \tag{18}$$

Accordingly, the optimal T_2 value for maximum w_{net} is obtained as follows:

$$T_{2,\text{opt}} = (T_3 T_1^k)^{\frac{1}{k+1}} = T_1 \tau^{\frac{1}{k+1}} = T_{4,\text{opt}}$$
(19)

Here it can be seen that the maximum w_{net} can be achieved when $T_{2,\text{opt}} = T_{4,\text{opt}}$. Accordingly, the following expression for the maximum w_{net} ($w_{\text{net,max}}$) can be obtained:

$$w_{\rm net,max} = c_v T_1 \left[\tau + k - (k+1)\tau^{\frac{1}{k+1}} \right]$$
(20)

Here, the optimal w_{net} value only changes depending on the T_3 temperature. Thus, the optimal thermal efficiency for $w_{net,max}$ can be expressed as follows:

$$\eta_{\rm th,opt} = 1 - k \frac{\tau^{1/(k+1)} - 1}{\tau - \tau^{1/(k+1)}}$$
(21)

2.3. Optimization of ε_2 for maximum p_{me}

The following equation can also be written for the variation of the previously expressed mean effective pressure with respect to the unknown value T_4 :

$$p_{\rm me} = \frac{p_1}{k-1} \frac{\tau \left(1 - \frac{T_4}{T_1}\right)^{-k} - k \left(\frac{T_4}{T_1} - 1\right)}{\frac{T_4}{T_1} \left(1 - \frac{1}{\tau} \frac{T_4}{T_1}\right)^{\frac{1}{k-1}}}$$
(22)

 $p_{\rm me}$ is an indicator of work potential independent of piston displacement for reciprocating engines. Now the following equation can be written for the variation of the mean effective pressure with respect to the unknown value ε_2 :

$$p_{\rm me} = \frac{p_1 \tau \varepsilon_2^k}{(k-1)(\varepsilon_2 - 1)} \left(\tau - \tau^{k-1} \varepsilon_2^{k^2 - k} - k \frac{\varepsilon_2^{1-k}}{\tau} + k \right)$$
(23)

Derivatives of the mean effective pressure expressions written here with respect to T_4 and ε_2 , respectively, are obtained as follows:

$$\frac{dp_{\rm me}}{dT_4} = -\frac{T_1 p_1 \left(\frac{T_4}{(1 - T_4/T_1)^k} - k(T_4/T_1 - 1)\right)}{(k - 1)T_4^2 \left(1 - \frac{T_4}{\tau T_1}\right)^{\frac{1}{k - 1}}} + \frac{p_1 \left(1 - \frac{T_4}{\tau T_1}\right)^{-\frac{k}{k - 1}} \left(\frac{\tau}{(1 - T_4/T_1)^k} - k\left(\frac{\tau}{T_1} - 1\right)\right)}{\tau T_4 (k - 1)^2} + \frac{T_1 p_1 \left(\frac{\tau k (1 - T_4/T_1)^{-k - 1}}{T_1} - \frac{k}{T_1}\right)}{(k - 1)T_4 \left(1 - \frac{T_4}{\tau T_1}\right)^{\frac{1}{k - 1}}} = 0$$
(24)

$$\frac{dp_{\rm me}}{d\varepsilon_2} = -\frac{\tau p_1 \varepsilon_2^k \left(-\tau^{k-1} \varepsilon_2^{k^2-k} - k \varepsilon_2^{1-k} + k + \tau\right)}{(k-1)(\varepsilon_2 - 1)^2} + \frac{\tau k p_1 \varepsilon_2^{k-1} \left(-\tau^{k-1} \varepsilon_2^{k^2-k} - k \varepsilon_2^{1-k} + k + \tau\right)}{(k-1)(\varepsilon_2 - 1)} + \frac{\tau p_1 \varepsilon_2^k \left(-\tau^{k-1} (k^2 - k) \varepsilon_2^{k^2-k-1} + \frac{k^2 - k}{\varepsilon_2^k}\right)}{(k-1)(\varepsilon_2 - 1)} = 0$$
(25)

Here the above equations are implicit in T_4 and ε_2 . Therefore, an equation solver or trial-and-error approach should be applied here. Thus, the ε_2 . value obtained by solving these two equations is determined as the value at which the maximum $p_{\rm me}$ ($p_{\rm me,max}$) is obtained. Once the ε_2 value has been determined, other performance parameters can also be calculated. In this study, Equations 24 and 25 were solved with the help of Matlab to determine $p_{\rm me,max}$.

3. CASE STUDY

For the case study, first of all, the classical thermodynamic example was solved. $T_{\text{max}} = 1961.923 \text{ K}$ was calculated for the assumptions $\varepsilon_2 = 12:1$, $q_1 = 1000 \text{ kJ/kg}$, $T_1 = 300 \text{ K}$, $p_1 = 100 \text{ kPa}$ and k = 1.005/0.718 = 1.4. Here, T_{H} temperature is assumed to be 2000 K for the second law calculations. The results obtained for the reference condition and other conditions are compared in Table 1. Here, ε_2 value is equivalent to the geometric compression ratio of Atkinson engines in hybrid electric vehicles used today. q_1 value was chosen as an average value in terms of ease of calculation.

The value of ε_2 , at which the maximum w_{net} ($w_{\text{net,max}}$) is obtained in an ideal Atkinson cycle operating in the temperature range of 300 K to 1961.923 K ($\tau = 6.54$), was calculated as $\varepsilon_{2,\text{opt}}^{w_{\text{net}}} = 6.54^{1.4/(1.4^2-1)} =$ 15.462 with the help of Equation 17. Accordingly, ε_1 was calculated as $\varepsilon_{1,\text{opt}}^{w_{\text{net}}} = 15.462^{1/1.4} = 7.07$ with the help of Equation 18. Thus, the maximum w_{net} was calculated as $w_{\text{net,max}} = 0.718 \times 300 [6.54 + 1.4 - (1.4 + 1) \times 6.54^{1/2.4}] = 580.139 \text{ kJ/kg}$ with the help of Equation 20. For the case where the maximum w_{net} was obtained, the η_{th} can be calculated as $\eta_{\text{th,opt}}^{w_{\text{net}}} = 1 - 1.4(6.54^{1/2.4} - 1)/(6.54 - 6.54^{1/2.4}) = 0.619$ with the help of Equation 21. Thus it was calculated as $T_{\text{m1}} = 1193.704 \text{ K}$, $T_{\text{m2}} = 481.977 \text{ K}$, $x_{\text{dest}} =$ 214.149 kJ/kg, $x_{\text{exp}} = 794.289 \text{ kJ/kg}$. For this case, η_{II} was calculated as $\eta_3 = 4623.599 \text{ kPa}$, $T_2 = T_4 =$ 656.105 K, $q_2 = 357.529 \text{ kJ/kg}$, $p_{\text{me}} = 329.392 \text{ kPa}$. For an ideal Atkinson cycle operating in the temperature range of 300 K to 1961.923 K, the ε_2 value at which the maximum p_{me} was obtained as a result of solving Equations 24 and 25 with the help of Matlab is determined as 31.063.

Table 1 is presented to compare of the calculated values for $w_{net,max}$ and $p_{me,max}$.cases. The variation of w_{net} and p_{me} with respect to ε_2 for constant T_{max} is shown in Figure 2. When w_{net} reaches its maximum value ($w_{net,max}$), p_{me} is not yet at its maximum value. For $p_{me,max}$ it is necessary to create ε_2 higher than $w_{net,max}$'s.Increasing ε_2 after $p_{me,max}$ is reached decreases both w_{net} and p_{me} . As can be seen in Figure 2, it is advantageous to choose the ε_2 range where $w_{net,max}$ and $p_{me,max}$ values are obtained for an engine running with an ideal Atkinson cycle.

Parameter	For $\varepsilon_2 = 12$	For <i>w</i> _{net,max}	For $p_{\rm me,max}$
$T_3 = T_{\max} (\mathrm{K})$	1961.923	<u>1961.923</u>	<u>1961.923</u>
$P_3 = P_{\max} (kPa)$	3242.307	4623.6	12278.58
<i>T</i> ₂ (K)	569.165	655.978	969.518
P_2 (kPa)	940.612	1545.92	6067.678
\mathcal{E}_1	4.958	7.070	18.775
<i>E</i> ₂	<u>12.000</u>	15.462	31.063
q_1 (kJ/kg)	<u>1000</u>	937.669	712.546
q_2 (kJ/kg)	428.252	357.529	197.129
W _{net}	571.748	580.139	515.417
P _{me} (kPa)	299.297	329.393	373.857
$\eta_{ m th}$	0.572	0.619	0.723
T_{4}/T_{2}	1.276	<u>1.000</u>	0.512
x_{dest} (kJ/kg)	275.341	214.149	88.173
$x_{\rm exp}$ (kJ/kg)	847.089	794.289	603.590
$\eta_{ m II}$	0.675	0.730	0.854
$T_{\rm m1}$ (K)	1125.45	1193.70	1409.868
$T_{\rm m2}$ (K)	481.978	455.155	390.046

Table 1. Comparison of calculated parameters



Figure 2. Optimization of ε_2 for w_{net} and p_{me}

The variation of w_{net} and p_{me} depending on T_4/T_2 when T_{max} is constant is shown in Figure 3. It was explained above that $T_2 = T_4$. should occur for w_{net} to be maximum. As seen here, $w_{\text{net,max}}$ should be $T_2 = T_4$ while $p_{\text{me,max}}$ should be $T_2 < T_4$. As seen in Figure 3, both w_{net} and p_{me} decrease when $T_4 > T_2$.



Figure 3. Optimization of T_4/T_2 for w_{net} and p_{me}

The variation of w_{net} and p_{me} with respect to η_{th} according to a constant T_{max} temperature is shown in Figure 4. As it is known, η_{th} is a function of ε_1 and ε_2 as expressed in Equation 7. Figure 4 shows that η_{th} is higher at $p_{me,max}$ when a comparison is made for $w_{net,max}$ and $p_{me,max}$ ". When evaluated in terms of thermal efficiency, it is more advantageous for the ε_2 value to be chosen between $w_{net,max}$ and $p_{me,max}$ and even close to the value from which $p_{me,max}$ is obtained for the ideal Atkinson cycle.



Figure 4. Optimization of η_{th} for w_{net} and p_{me}

Although it shows that it is necessary to approach the $p_{me,max}$ value for high thermal efficiency, this may not be possible in reality. Because the ideal compression ratio of today's spark ignition engines varies between 8 to 11 (Ferguson and Kirkpatrick, 2015). Accordingly, the variation of ε_1 and η_{th} with respect to ε_2 according to a constant T_{max} is shown in Figure 5. Here, ε_2 varies in the range of 17.621 to 22.828, corresponding to the ideal ε_1 value in the range of 8 to 11. However, the ideal ε_1 region is closer to the condition in which $w_{net,max}$ is obtained, not the condition in which $p_{me,max}$ is obtained. Therefore, the maximum values of η_{th} and p_{me} are also limited due to the ideal ε_1 region.



Figure 5. Variation of ε_1 and η_{th} compared to ε_2

The variation of T_{m1} and T_{m2} with respect to η_{th} according to a constant T_{max} is shown in Figure 6. T_{m1} and T_{m2} are important in expressing the ideal Atkinson cycle as the Carnot cycle equivalent. Here, as expected, $T_{m1} - T_{m2}$ value increases as we approach from the $w_{net,max}$ condition to the $p_{me,max}$ condition, that is, η_{th} increases.



Figure 6. Variation of T_{m1} and T_{m2} compared to ε_2

The variation of x_{dest} and x_{exp} with respect to ε_2 according to a constant T_{max} is shown in Figure 7. Here, as we approach from $w_{net,max}$ condition to $p_{me,max}$ condition, the $x_{exp} - x_{dest}$ value increases, that is, η_{II} increases.



Figure 7. Variation of x_{dest} and x_{exp} compared to ε_2

In Figure 8, the relationship between p_{me} and w_{net} is shown for both constant T_{max} and constant q_1 . Here p_{me} and w_{net} tend to increase continuously for constant q_1 . However, in the case of constant T_{max} , there are situations where p_{me} and w_{net} are maximum. For this reason, it is advantageous to optimize the compression ratio for situations where the p_{me} or w_{net} values are maximum while designing the engine.



Figure 8. Comparison of p_{me} and w_{net}

4. CONCLUSION REMARKS

In this study, a case study was conducted using the equations obtained for the maximum specific net work and maximum mean effective pressure for the ideal Atkinson cycle. Considering the compression ratio values of today's engines, it has been seen that these values are closer to the compression ratio at which the maximum specific net work is achieved. However, when the compression ratio is optimized for conditions where the maximum mean effective pressure is obtained, it has been observed that the thermal efficiency also increases. However, in practice, it has also been seen that optimizing today's engines to achieve maximum mean effective pressure is not appropriate due to the fuel properties used. For this reason, it has been seen that it is more valid to choose the compression ratio of Atkinson cycle engines according to the maximum specific net work. Although a solution is discussed in this study with the focus of the ideal Atkinson cycle, this presented method can also be used for other gas power cycles. In addition, the model has been developed for the ideal conditions of the cycle. This model can be reformulated for realistic engine cycles by taking into account parameters such as irreversibility, friction loss and combustion efficiency.

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