



A Two-Phase Approach for Reliability-Redundancy Optimization of a Communication Satellite

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Highlights

- Reliability optimization of a communication satellite consisting of active and passive redundancy.
- Minimization of the cost by considering reliability requirements subject to related constraints.
- A two-phase approach is offered which transforms the non-linear problem to an integer model.
- Optimal redundancy levels could be obtained to achieve an efficient system design.

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Abstract

The development and launch of communication satellite projects pose significant challenges and costs. The expenses can range from several hundred million dollars, contingent on factors such as mission objectives, satellite system size and complexity including the launch vehicle, and ground infrastructure. Satellites must be designed to withstand harsh conditions in space, such as the extreme temperatures, radiation, and other hazards, while delivering reliable communication services to its users. However, once a satellite is launched, physical maintenance interventions become infeasible in the event of technical problems. Thus, reliability is a critical aspect for these expensive systems.

This study aims to minimize the cost of a high-tech communication satellite by addressing design considerations that meet customer reliability requirements without exceeding power and redundant equipment limits. To achieve this goal, we propose an integer non-linear programming model in this research. To solve the satellite design problem, we adopt a two-stage solution approach. Conventional industrial practices in satellite design often involve iterative attempts to determine the redundancy level of onboard units based on customer reliability requirements. These processes rely heavily on the experience of design engineers who evaluate a limited number of alternatives to determine the number of redundant units, resulting in sub-optimal outcomes. In contrast, our proposed approach systematically handles the problem and yields optimal results. Our findings demonstrate that the proposed two-phase approach can achieve optimal redundancy levels within seconds.

1. INTRODUCTION

In the last century, space has been a source of ambition and inspiration for humankind, with aspiring projects that have revealed revolutionary advancements and spin off technologies. One such advancement was the development and proliferation of satellites in various types, aimed at realizing ambitious goals related to space. The reliability of a satellite system plays a critical role in achieving the desired expectations for space projects through cutting-edge technological satellites, where maintenance and repair are not possible following the launch of a spacecraft. Reliability is defined as the ability of a system or component to perform its functions under defined conditions and at the desired performance level for a specified period of time [1-3].

The idea of communication satellites came to the fore with Arthur C. Clarke's article published in the *Wireless World* magazine in 1945 [4]. Communication Satellites are used in locations where terrestrial lines cannot reach, serving as a continuation of intercontinental cable systems and in civilian areas such as

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television and radio broadcasting [5]. In television broadcasting, the signals sent from the center to the satellite are redistributed to large areas after their frequencies are changed and signal levels are strengthened [6]. These types of satellites are located in the geosynchronous orbit with the same rotational speed as the Earth and situated at approximately 35.786 km altitude.

Typically, the budget for a medium-sized communication satellite is around several hundred million dollars and these satellites are launched into space with significant costs based on their weight [7]. Once satellites are launched into space, there is no possibility of maintenance intervention on the ground in the event of a malfunctions, as they face harsh space environments with threats to the satellite reliability, such as radiation, high-temperature differences, solar activities, and meteorites. An established strategy for preventing loss of functionality is the use of redundancy philosophy and reliability approaches considered during the design [8]. Therefore, reliability has long been recognized as a critical attribute for space systems and an essential parameter in spacecraft design and optimization [9].

Each piece of equipment that is placed onboard in excess is called the redundant unit and it enhances reliability as it can be used in case of failure. However, these redundant units add volume, mass, power requirements, and the need for additional cabling, thereby increasing both cost and the complexity of the overall system. To mitigate this issue, it is possible to design a cost- and resource-efficient satellite by optimizing the reliability and redundancy of the system.

The reliability of a system can be improved through two principal methods; increasing the individual reliability of components and/or adding redundant components to the system [10]. Incorporating redundant units in to a system is more cost-effective than improving the reliability of individual components. Therefore, in the literature, the problem referred to as the Redundancy Allocation Problem (RAP), which has received significant attention in recent decades [11-19]. However, improving reliability by adding additional components will increase the cost of the system [20].

In this study, we consider a communication satellite composed of several subsystems with active and passive redundancy schemes onboard. Our aim is to determine the number of active/passive redundant units in each subsystem to minimize the cost of the system while considering a set of constraints related to the subsystems. The developed model is a non-linear integer programming model. Unlike most studies in the literature, we offer an exact approach for solving the problem. The proposed method suggests a systematic approach to the satellite design process which is typically reliant on manual iterative attempts to determine the onboard redundancy levels. Section 2 provides a literature review followed by the problem definition and the mathematical model in section 3. The proposed two-phase approach is explained in section 4. We discuss the results from a case study of a mid-size communication satellite design in section 5. Finally, we present concluding remarks and future research directions in section 6.

2. LITERATURE REVIEW

The reliability and redundancy allocation problems have been addressed in the literature using various methods; including interval optimization [21], geometric programming [22], differential dynamic programming method [23-25], as well as exact methods such as Lagrangian relaxation and dynamic programming [26], branch-and-bound [27,28], and cutting plane methods based on a linear integer programming relaxation and successive application of the simplex algorithm [29]. Other approaches including lexicographic search and upper bound based algorithm [30], an improved surrogate constraint method [31], functional evaluations and a limited search close to the boundary of resources based mixed integer programming algorithm [32]. Since this problem is known to be NP-Hard [33], several metaheuristic algorithms have been proposed, such as Simulated Annealing (SA) [34-36], Genetic Algorithms (GA) [37-39], Simplified Swarm Optimization (SSO) [40], and Tabu Search (TS) [41]. For more comprehensive classification and survey about past studies on reliability optimization models and solution approaches, interested reader can refer to the review studies published by Kuo & Prasad, Tillman et al, and Misra [42-46] as well as relatively recent studies by Soltani, Twum, and Aspinwall [47,48].

Not only the number of exact approaches for the RAP limited but there are also only a few studies in the literature that focus on satellite design and reliability optimization. In [49]; the minimization of total launch mass and the maximization of spacecraft overall reliability as a multi-objective optimization problem is addressed using Genetic Algorithms. The RAP of phase mission systems with mixed redundancy strategy using non-exponential components is studied and the optimization of a spacecraft propulsion subsystem is illustrated through the proposed method in [50]. The power amplifiers redundancy scheme of a communication satellite payload module with an analytical approach under different failure rate has been studied in [51]. A genetic algorithm with Monte Carlo sampling for probabilistic reliability-based design optimization of satellite systems is presented in [52]. Extended statistical analysis of satellite reliability is provided in [53] which investigating the reliability of satellite subsystems with Weibull distributions using the maximum likelihood estimation approach. Table 1 provides a summary of studies in the literature related to satellite design and reliability optimization.

Table 1. Summary Table of Studies on Reliability Optimization of Satellite Design

Author, Year	Objectives	Subject to	Method
Hassan & Crossley, 2003	<ul style="list-style-type: none"> - Minimization of mass - Maximization of reliability 	<ul style="list-style-type: none"> - Solar panel length, radiator panel height, max allowable lift mass, reliability of the launcher, satellite payload and spacecraft 	<ul style="list-style-type: none"> - Genetic Algorithm
Hassan & Crossley, 2008	<ul style="list-style-type: none"> - Minimization of mass 	<ul style="list-style-type: none"> - Solar panel length, radiator panel height, max allowable lift mass, reliability of the launcher, satellite payload and spacecraft 	<ul style="list-style-type: none"> - Genetic Algorithm with Monte Carlo sampling
Nefes et al, 2018	<ul style="list-style-type: none"> - Maximization of reliability - Minimization of cost 	<ul style="list-style-type: none"> - Reliability of power amplification - System level reliability 	<ul style="list-style-type: none"> - Analytical Method in Matlab
Li, X. et al, 2020	<ul style="list-style-type: none"> - Minimization of mass - Maximization of reliability 	<ul style="list-style-type: none"> - Weight limit - System level reliability 	<ul style="list-style-type: none"> - Semi-Markov Process - Genetic Algorithm
This Study	<ul style="list-style-type: none"> - Minimization of cost 	<ul style="list-style-type: none"> - System level reliability - Max allowable number of redundant units - Limits of redundant units for each subsystem - Power consumption of redundant units 	<ul style="list-style-type: none"> - Exact Approach

To the best of our knowledge, there is no exact approach for the reliability and redundancy optimization of a communication satellite system in the literature, considering the given restrictions. This is likely due to the fact that mathematical models involving reliability calculations are mostly non-linear. As a result, researchers tend to resort to metaheuristic algorithms to solve these problems. The model we propose for a communication satellite is also a non-linear model. To facilitate the solution of the model, we have reformulated the problem using a two-phase approach. In the next section, we introduce the non-linear model and present our proposed solution approach.

3. PROBLEM DEFINITION

Satellites are operated in space under harsh environmental conditions including radiation, solar activities, meteorites, and high temperature differences. Once a satellite is launched into space, there is no possibility of physical intervention from the ground in case of any malfunctions. Therefore, to minimize the loss of

functionality, redundant units and reliability limits are taken into account during the design phase of a satellite.

A typical communication satellite comprises various subsystems including two major modules; the communication payload module (or communication modules, which include antennas and a repeater) and the main platform or service module (comprising several subsystems) which supports the payload module. Each of these subsystems consists of equipment chains connected in parallel, following a specified redundancy scheme to meet the system-level reliability requirements. The repeater consists of multiple electronic equipment that perform various functions on the transmission signals. It is made up of several channels, also called transponders, which are dedicated to sub-bands within the overall payload frequency band [54]. The block diagram of a repeater channel typical includes various passive and active units, such as switches, amplification units (Electronic Power Conditioner-EPC and Travelling Wave Tube -TWT), attenuators and High Power Isolators (HPI) connected in series as depicted in Figure 1.

The main platform module consists of various subsystems, including the Attitude and Control Subsystem (AOCS), Satellite Command and Control Subsystem (SCS), Electrical Power Subsystem (EPS), Data Handling Subsystem (DHS), Propulsion Subsystem (PS), Thermal Control Subsystem (TCS), Structural Subsystem (SS) and Telemetry, Telecommand and Ranging Subsystem (TTCR) [55]. However, these subsystems are not considered in this study as there is limited potential for improvement in terms of redundancy allocation. In most satellite designs, high levels of redundancy, such as one-to-one redundancy, are already ensured due to the limited number of equipment placed in the subsystem architecture.

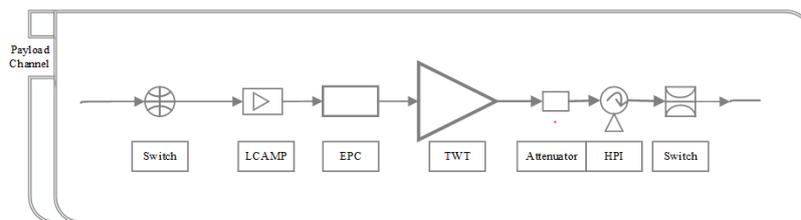


Figure 1. Communication Satellite Payload Channel

The communication payload module we are focusing on consists of numerous active components with high failure rates and relatively more intense power consumption and heat dissipation. Unlike the platform subsystems (AOCS, SCS, EPS, DHS, etc.), maintaining high levels of redundancy in the payload subsystem is not feasible due to the significant number of equipment involved. Instead, the redundancy scheme is provided through utilization of a spare pool in the event of equipment failure. The reliability block diagram of the considered mid-size communication satellite payload module, which is composed of three subsystems in a k-out-of-n configuration (or simply k:n) is shown in Figure 2. In a k-out-of-n configuration, the system functions properly if any k or more components ($k \leq n$) are functioning correctly [56].

There are three types of k:n redundant systems in the literature that are actively used in complex systems; hot standby, warm standby, and cold standby [57]. In satellite system architecture, the hot standby and warm standby types are generally utilized as active redundancy and passive redundancy modes respectively, based on operational mode and criticality of the subsystems. In hot standby systems, all n components are in an active operating state with the load distributed simultaneously among them and the failure rate of the hot standby component is the same as that of the main unit. In the warm standby systems, only k units are in active mode and take over the load while the remaining components (n-k of them) which are in online state and do not share the load. In the warm standby system, the failure rate of the standby unit is lower than that of the main unit. In the cold standby systems, all of the redundant components are in an offline state and need to be powered up and switched to the operating mode when required. Cold standby units are typically used for repairable systems, where backup spare parts serve as cold standby units. In case of a failure, the defective component is replaced with the spare unit to restore system operation promptly [56]. For both the payload and main platform subsystems mentioned above, hot standby architectures are considered for active units, while warm standby architectures are considered for passive units. During the design process the payload channels with active redundancy may be prioritized for high-priority

transmissions, while those with passive redundancy are used for standard communication. The specific power consumption and reliability figures are utilized in the calculations depending on the mode of active or passive redundancy.

In this study, we optimize the number redundant of active/ passive channels (units) in each subsystem. The first subsystem (referred to as Mission-1 in Figure 2) consists of both active and passive redundant units. The second subsystem (Mission-2 in Figure 2) and the third subsystem (B1 Subsystem in Figure 2) only consider active redundant units. In a typical mid-size communication satellite, the payload module can accommodate around 20 to 30 nominal payload channels. Within this context, the contribution of the rest of the platform subsystem is considered as a constant.

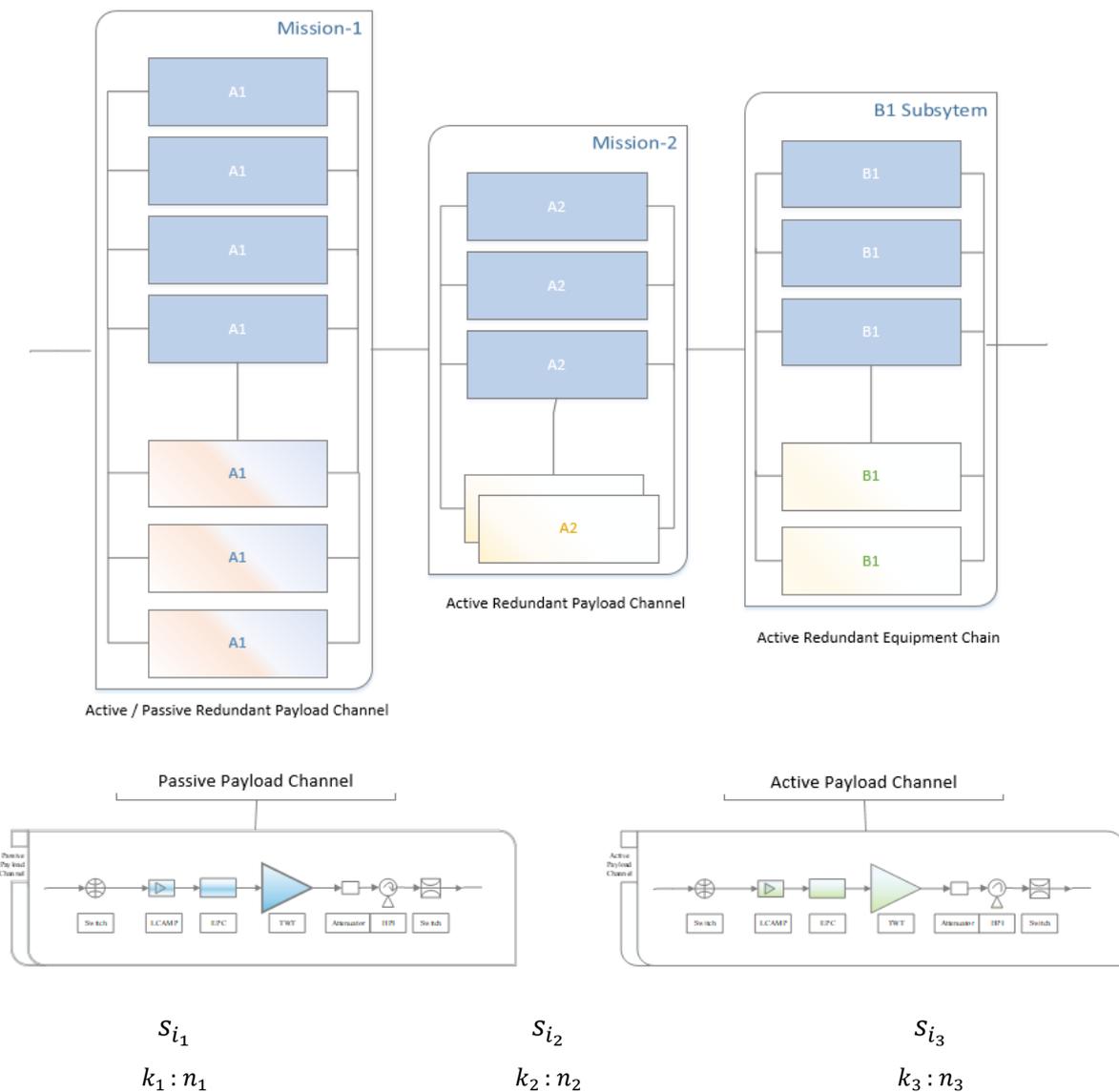


Figure 2. *k-out-of-n* Satellite Subsystems Reliability Block Diagram

The aim of the following model is to minimize the total cost of redundant equipment in subsystems of a communication satellite while ensuring compliance with reliability level requirements, power consumption limitations, and redundancy level constraints.

Parameters:

c_i	; cost of redundant equipment i
k_i	; minimum number of equipment required for subsystem i
n_i	; total number of equipment in subsystem i , $n_i = k_i + x_i$
n_i^l	; minimum number of redundant equipment
n_i^u	; maximum number of redundant equipment
n_{\max}	; maximum number of redundant equipment in the system
P_i	; power consumption of equipment i (Watt)
P_{red}	; maximum allowable power consumption of redundant equipment (Watt)
R_i	; reliability of subsystem i
R_{sys_min}	; minimum required reliability value of system

Decision Variables:

$x_i = n_i - k_i$; number of redundant equipment in subsystem i

$$\min \quad Z = g(x) = \sum_{i=1}^s x_i c_i \quad (1)$$

$$\text{st.} \quad \prod_{i=1}^s \sum_{j=k_i}^{k_i+x_i} \binom{k_i+x_i}{k_i} R_i^j (1-R_i)^{k_i+x_i-j} \geq R_{sys_min} \quad (2)$$

$$\sum_{i=1}^s x_i \leq n_{\max} \quad (3)$$

$$n_i^l \leq x_i \leq n_i^u, \quad \forall i \quad (4)$$

$$\sum_{i=1}^s x_i P_i \leq P_{red} \quad (5)$$

$$x_i \geq 0 \text{ and integer} \quad (6)$$

The objective function given in Equation (1) aims to minimize the total cost of the system. Constraint (2) ensures that the reliability of subsystems exceeds a specified threshold value. Constraint (3) limits the number of redundant equipment that can be accommodated. Constraint set (4) defines the lower and upper limits for the number of redundant equipment in each *subsystem* i . Constraint (5) restricts the power consumption of redundant units. The decision variables are defined within the domain specified by Constraint (6). The presented model is a non-linear model due to the Constraint (2). To address this, we propose a two-phase approach to transform the model into a manageable integer programming problem.

4. THE TWO-PHASE APPROACH

In the first phase of the proposed approach, we define a set of possible equipment configurations for each subsystem which includes both active and passive components. Each configuration consists of different numbers of redundant equipment. The specific configurations for the satellite subsystems shown in Figure 2 are presented in section 4 as part of the case study. The reliability functions for the active and passive

channels are represented by Equation (7) and Equation (8) respectively. In k:n hot standby (active) redundant systems, the system reliability can be modeled using the binomial distribution and expressed as follows [56]:

$$R(t) = \sum_{i=k}^n C \binom{n}{i} (1 - R(t))^{n-i} (R(t))^i \quad (7)$$

where $R(t) = e^{-\lambda t}$ denotes reliability with constant failure rate λ considered throughout service life of a satellite [58], given that the components used are identical and independent, $R_1(t) = R_2(t) = \dots = R_n(t)$.

In k:n warm standby (passive) redundant systems, where all components are assumed to be independent, identically distributed, and each component follows an exponential distribution with the parameter λ_o representing the failure rate for active operational equipment and the parameter λ_d representing the failure rate for stationary equipment derived a closed-form expression for the system reliability function [59]. This expression is given as follows [60];

$$R(t) = \frac{1}{(n-k)! \lambda_d^{n-k}} \sum_{i=0}^{n-k} (-1)^i \binom{n-k}{i} [\prod_{j=0, j \neq i}^{n-k} (k\lambda_o + j\lambda_d)] e^{-(k\lambda_o + i\lambda_d)t} . \quad (8)$$

When $\lambda_d = \lambda_o = \lambda$, the passive redundant system formula given in Equation (8) reduces to the formula given in Equation (7).

In the second phase, the optimal configurations are selected from a generated set composed of possible equipment configurations using 0-1 integer programming model presented below. Since the reliability of each configuration is calculated in the previous step, these reliability figures $R_{i_1}, R_{i_2}, R_{i_3}$ for each configuration is supplied to the second step as an input.

Parameters:

$m_{i_1}, m_{i_2}, m_{i_3}$; number of redundant units in equipment chains $i_1, i_2,$ and i_3
$nb_{i_1}, nb_{i_2}, nb_{i_3}$; number of redundant units available in subsystem $s_{i_1}, s_{i_2},$ and s_{i_3}
$n_{i_1}^l, n_{i_1}^u, n_{i_2}^l, n_{i_2}^u, n_{i_3}^l, n_{i_3}^u$; lower and upper limits of number of redundant units in subsystem $s_{i_1}, s_{i_2},$ and s_{i_3}
n_{max}	; maximum number of total redundant units in the system
$P_{i_1}, P_{i_2}, P_{i_3}$; power consumption of equipment chains $i_1, i_2,$ and i_3 (Watt)
P_{red}	; maximum allowable power consumption of redundadant units (Watt)
$R_{i_1}, R_{i_2}, R_{i_3}$; reliability value of subsystems $s_{i_1}, s_{i_2},$ and s_{i_3}
R_{sys_min}	; minimum required reliability value of subsystem

Decision variables:

$x_{i_1}, y_{i_2}, z_{i_3}$; selection of a configuration or not
 $i_1 = 1, 2, 3 \dots nb_{i_1}$ $i_2 = 1, 2, 3 \dots nb_{i_2}$ $i_3 = 1, 2, 3 \dots nb_{i_3}$

$$\min \quad Z = g(x) = \sum_{i_1=1}^{nb_{i_1}} x_{i_1} c_{i_1} + \sum_{i_2=1}^{nb_{i_2}} y_{i_2} c_{i_2} + \sum_{i_3=1}^{nb_{i_3}} z_{i_3} c_{i_3} \quad (9)$$

$$st. \left[\sum_{i_1=1}^{nb_{i_1}} x_{i_1} R_{i_1} \right] \cdot \left[\sum_{i_2=1}^{nb_{i_2}} y_{i_2} R_{i_2} \right] \cdot \left[\sum_{i_3=1}^{nb_{i_3}} z_{i_3} R_{i_3} \right] \geq R_{sys_min} \quad (10)$$

$$\sum_{i_1=1}^{nb_{i_1}} x_{i_1} m_{i_1} + \sum_{i_2=1}^{nb_{i_2}} y_{i_2} m_{i_2} + \sum_{j=1}^{nb_j} z_{i_3} m_{i_3} \leq n_{max} \quad (11)$$

$$n_{i_1}^l \leq \sum_{i_1=1}^{nb_{i_1}} x_{i_1} m_{i_1} \leq n_{i_1}^u \quad (12)$$

$$n_{i_2}^l \leq \sum_{i_2=1}^{nb_{i_2}} y_{i_2} m_{i_2} \leq n_{i_2}^u \quad (13)$$

$$n_{i_3}^l \leq \sum_{i_3=1}^{nb_{i_3}} z_{i_3} m_{i_3} \leq n_{i_3}^u \quad (14)$$

$$\sum_{i_1=1}^{nb_{i_1}} x_{i_1} m_{i_1} P_{i_1} + \sum_{i_2=1}^{nb_{i_2}} y_{i_2} m_{i_2} P_{i_2} + \sum_{i_3=1}^{nb_{i_3}} z_{i_3} m_{i_3} P_{i_3} \leq P_{red} \quad (15)$$

$$\sum_{i_1=1}^{nb_{i_1}} x_{i_1} = 1 \quad (16)$$

$$\sum_{i_2=1}^{nb_{i_2}} y_{i_2} = 1 \quad (17)$$

$$\sum_{i_3=1}^{nb_{i_3}} z_{i_3} = 1 \quad (18)$$

$$x_{i_1} \in \{0,1\} \quad \forall nb_{i_1} \quad (19)$$

$$y_{i_2} \in \{0,1\} \quad \forall nb_{i_2} \quad (20)$$

$$z_{i_3} \in \{0,1\} \quad \forall nb_{i_3} \quad (21)$$

Similar to the original form of the problem given by Equation (1) to Equation (6), the aim of the proposed model is to minimize the total cost of the system in Equation (9). The Constraint (10) ensures that the reliability of subsystems should be greater than a minimum threshold value. In Constraint (10), R_{i_1} is calculated by Equation (7) and Equation (8) whereas R_{i_2} and R_{i_3} are calculated by Equation (7). The Constraints (11) through (14) maintain the allowable thresholds for number of redundant equipment in the system. With the Constraint (15), the available power resources onboard are secured for the redundant equipment. The Constraints (16) through (18) ensure that a configuration should be selected for each subsystem. Finally, the Constraints (19) through (21) shows whether the presented configuration alternative is selected or not.

The constraint (10) represents the minimum system-level reliability requirement, but it is in a complex non-linear form, making it difficult to solve directly. To simplify the compound form of the constraint, a decomposition approach is used. The decomposition approach is about taking the logarithms of the terms on the left hand side of Equation (10) in the context of separable programming [61]. The resultant constraint is shown in Equation (22);

$$\log \left[\sum_{i_1=1}^{nb_{i_1}} x_{i_1} R_{i_1} \right] + \log \left[\sum_{i_2=1}^{nb_{i_2}} y_{i_2} R_{i_2} \right] + \log \left[\sum_{i_3=1}^{nb_{i_3}} z_{i_3} R_{i_3} \right] \geq \log R_{\text{sys_min}} \quad (22)$$

The resultant model is solved using GAMS Dicopt Solver. The details about how to code logarithm in GAMS could be found in GAMS Documentation Center.

5. RESULTS

In order to demonstrate the model performance, a case study with scaled values of a communication satellite is solved with the proposed two-phase approach. The selected subsystem configuration presented in section 3, namely the Mission-1, the Mission-2 and the B1 subsystems' redundancy scheme is configured as a k-out-of-n redundant systems which are commonly found in a typical communication satellites. The reliability figures of the units are derived from actual Failure-In-Time (FIT) values which represents the number of failures per billion hours of operation for each unit. The number of units in each subsystem is determined based on typical practices in the industry for a midsize communication satellite. It is important to note that the design life time of a typical geosynchronous communication satellite is considered as 15 years which is converted to hours and represented as the (t) parameter in the equations used for the case study calculations.

In Table 2, we examined various configurations ranging from 20:21 to 20:30 payload channels for Mission-1. These configurations encompass both active and passive redundant units. Similarly, in Table 3 we explored configurations featuring 6:7 to 6:12 payload channels for the Mission-2 with active redundancy scheme. Lastly, in Table 4 we investigated configurations of 4:5 to 4:8 for the B1 subsystem also employing an active redundancy scheme. Consequently, we obtained a total of 30 alternative configurations. For each of these predefined configurations, the reliability (R_i) values are calculated as well as the cost (c_i) and power (P_i) values. The reliability scores for active and passive channels and equipment were determined by using the Equation (7) and Equation (8) respectively.

In Tables 2, 3 and 4; associated cost and reliability for each alternative configuration considering various number of redundant equipment are presented.

Table 2. $k:n$ Redundant s_{i_1} Subsystem (standart channels) Model Parameters, $R_{\text{equipment}}=0.8884$

Conf.	Active (A)/ Passive (P)	k	n	m_{i_1} (red. eq.)	R_{i_1} (reliability)	c_{i_1} (cost, kUSD)	P_{i_1} (power, W)
1	P	20	21	1	0.3144	750	1
2	P	20	22	2	0.5753	1500	2
3	P	20	23	3	0.7820	2250	3
4	P	20	24	4	0.9053	3000	4
5	P	20	25	5	0.9646	3750	5
6	P	20	26	6	0.9884	4500	6
7	P	20	27	7	0.9966	5250	7
8	P	20	28	8	0.9991	6000	8

9	P	20	29	9	0.9998	6750	9
10	P	20	30	10	1.0000	7500	10
11	A	20	21	1	0.3031	750	50
12	A	20	22	2	0.5484	1500	100
13	A	20	23	3	0.7492	2250	150
14	A	20	24	4	0.8780	3000	200
15	A	20	25	5	0.9470	3750	250
16	A	20	26	6	0.9791	4500	300
17	A	20	27	7	0.9924	5250	350
18	A	20	28	8	0.9975	6000	400
19	A	20	29	9	0.9992	6750	450
20	A	20	30	10	0.9998	7500	500

Table 3. $k:n$ Redundant s_{i_2} Subsystem (high priority channels) Model Parameters, $R_{equipment}=0.8943$

Conf.	Active(A)/ Passive(P)	k	n	m_{i_2} (red. eq.)	R_{i_2} (reliability)	C_{i_2} (cost, kUSD)	P_{i_2} (power, W)
1	A	6	7	1	0.8359	750	50
2	A	6	8	2	0.9560	1500	100
3	A	6	9	3	0.9898	2250	150
4	A	6	10	4	0.9979	3000	200
5	A	6	11	5	0.9996	3750	250
6	A	6	12	6	0.9999	4500	300

Table 4. $k:n$ Redundant s_{i_3} Subsystem (B1 Subsystem) Model Parameters, $R_{equipment}=0.9677$

Conf.	Active(A)/ Passive(P)	k	n	m_{i_3} (red. eq.)	R_{i_3} (reliability)	C_{i_3} (cost, kUSD)	P_{i_3} (power, W)
1	A	4	5	1	0.99020	200	10
2	A	4	6	2	0.99937	400	20
3	A	4	7	3	0.99996	600	30
4	A	4	8	4	1.00000	800	40

The reliability figures for each configuration were calculated using the formulas provided in Equation (7) and Equation (8). This data was then input into GAMS to solve the model with Equation (9) subjected to the Constraints (11) through (21) and Constraint (22). The model is solved in 1.83 seconds using an Intel Core i7-7600 CPU @ 2.80 GHz and an optimal solution was obtained. The results of the optimization are presented in Table 5 which shows that the model selected 6 passive redundant channels for s_{i_1} , 4 active redundant channels for s_{i_2} , and 2 active redundant equipment for s_{i_3} subsystems.

Table 5. The Results of Redundancy Scheme

Subsystem	Number of Redundant Equipment	Active(A)/ Passive(P) Redundancy
s_{i_1}	6	P
s_{i_2}	4	A

s_{i_3}	2	A
Objective Function Value (kUSD)	7900	

Based on the solution presented in Table 5, the total power consumption of the redundant units is 226 Watts and the reliability of subsystems is 0.9857. The algorithm indicates that these results are reasonable and consistent with the industrial practices, as they align with expert judgements. This approach provides an optimal redundancy solution using a deterministic method.

6. CONCLUSION

This study focuses on the reliability optimization of a geosynchronous communication satellite, incorporating both active and passive redundancy schemes while considering various design constraints. The objective is to achieve a cost-efficient system design by determining optimal levels redundancy that satisfy reliability requirements and power consumption constraints of the units. Notably, this work introduces the first implementation of an exact approach to address the redundancy allocation problem of a communication satellite.

First, a non-linear model is introduced for the problem which is known as NP-hard in the RAP literature. Next, a novel two-phase approach is developed to obtain a simple equivalent model. In this approach, the reliability values of defined plausible configurations including active and passive units are calculated in the first phase and the optimum configurations are selected from the generated set using an integer programming model in the second phase. Although the resulting mathematical model from the two-phase approach remains non-linear due to the reliability constraint, the non-linearity is relatively manageable. Through the utilization of a decomposition approach, a simplified model is obtained. This model is solved using GAMS.

To evaluate the performance of the proposed model, a numerical case study was conducted on a sample configuration inspired by a typical mid-size communication satellite system. This configuration included of two payload missions (Mission-1 and Mission-2) with active and passive redundancy schemes, as well as an additional active equipment subsystem (B1 subsystem) with scaled real values of parameters. The obtained results demonstrated that the proposed two-stage approach yields optimize and reasonable results that are in line with practical applications in the industry when compared to the subsystem configurations of existing satellites in similar scales. The optimal redundancy levels were obtained within seconds using this proposed two-phase approach.

The proposed methodology suggests a systematic approach to find a cost-effective solution satisfying reliability requirements within a reasonable timeframe. In general, this systematic approach can be effectively implemented to achieve a cost-effective system design while taking reliability requirements into account, especially in large-scale and intricate engineering projects where the performance and reliability of the system must be guaranteed over extended periods. It also paves the way for sensitivity analysis in complex systems' design under given constraints by considering reliability value versus total cost of the system. It provides a framework to explore the trade-off between reliability values and the total cost of the system, allowing for informed decision-making and optimization. This capability enhances the understanding of the system's behavior and the impact of reliability considerations on the overall cost-effectiveness.

Overall, the proposed methodology offers a valuable systematic approach that can be applied to various engineering projects, enabling the attainment of cost-effective designs while satisfying reliability requirements and facilitating sensitivity analysis.

Future work in this field could involve the application of a multi-objective optimization approach to address the problem of conflicting objectives in satellite system design. By considering multiple objectives

simultaneously, such as cost, reliability, power consumption, and other relevant factors, a more comprehensive and well-balanced solution can be achieved.

Additionally, it would be beneficial to expand the scope of the problem formulation by incorporating additional design constraints specific to satellite systems. These constraints could include factors such as weight limitations, communication bandwidth requirements, thermal constraints, and operational constraints. Integrating these constraints into the optimization model would provide a more realistic representation of the design problem and further enhance the practicality of the proposed methodology.

Furthermore, future research could explore advanced modeling techniques, such as stochastic programming or probabilistic approaches, to capture uncertainties inherent in satellite system design. By incorporating probabilistic models, the impact of uncertainties, such as component failure rates or environmental factors, can be quantified, leading to more robust and reliable designs.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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