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Axisymmetric Finite Cylinder with Rigid Ends and a Circumferential Edge Crack

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ÖZET

Bu çalışmada doğrusal elastik ve izotropik malzemeden imal edilmiş, çevresel bir kenar çatlağı içeren rijit uçlu sonlu uzunlukta eksenel simetrik bir silindir incelenmiştir. Sonlu silindir problemine ulaşmak için, halka şeklinde iç çatlak ve iki dairesel şekilli enklüzyon içeren sonsuz bir silindirden yola çıkılmıştır. Navier denklemleri Fourier ve Hankel dönüşümleri kullanılarak çözülmüştür. Formülasyon tekil integral denklemlerine indirgenmiş ve Gauss-Lobatto ve Gauss-Jakobi integrasyon formülleri ile doğrusal cebrik denklem takımlarına dönüştürülmüştür. Çatlağın kenarlarındaki ve silindirin çevresi etrafındaki gerilme yığılma faktörleri hesaplanmıştır.

Anahtar Kelimeler: Kenar Çatlak, Sonlu Silindir, Gerilme Yığılma Faktörü, Rijit Enklüzyon

ABSTRACT

An axisymmetric linearly elastic and isotropic finite cylinder with rigid ends and a circumferential edge crack subjected to axial tension is considered. Finite cylinder problem is obtained from an infinite cylinder containing an internal ring-shaped crack and two penny-shaped rigid inclusions. Navier equations are solved by using Fourier and Hankel transforms. Formulation is reduced to three singular integral equations which are converted to a system of linear algebraic equations with the aid of Gauss-Lobatto and Gauss-Jacobi integration formulas. Stress intensity factors at the edges of crack and around the corner of the cylinder are calculated.

Keywords: Edge Crack, Finite Cylinder, Stress Intensity Factor, Rigid Inclusion.

1. INTRODUCTION

Discontinuities may create excessive stress concentrations in structural elements and machine parts. Machine elements with discontinuities are commonly used in many areas. These discontinuities may occur in the regions of sharp edges, openings, cracks or inclusions as well as at the regions where the geometric properties of the cross section changes. Presence of discontinuities may severely affect the load resisting capacity of the element and hence the system. Distribution of stresses around these discontinuities may be very complicated. In such regions, stress distributions may be calculated in terms of the stress intensity factors.

Machine elements with large probability of containing singularities are very important in fracture mechanics. Finite cylinders are among these elements. Stresses in the vicinity of the crack and inclusion tips alternate with singularity, regardless of the configuration of the cracked element. In general, these sorts of problems may be studied by numerical and analytical methods based on the solution of corresponding partial differential equations. The assumption of linear elastic material allows to the superposition of the stress and displacements. This superposition principle provides the solution for complex finite cylinder arrangements analytically by using the combination of simple cases. Solutions for finite cylinder problems containing edge cracks and penny-shaped inclusions can be found in the literature.

Chang (1985) obtained the general solution of the stress intensity factor of a finite cylinder containing a concentric penny-shaped crack under torsion. The general solution has been obtained by using Hankel transform and Fourier series. It has been proved that the solutions of a penny-shaped crack in an infinite long cylinder and in a circular plate of infinite radius may be derived from the general solution presented in this work. Zhang (1988) considered the problem of concentric penny-shaped crack in a finite orthotropic cylinder under torsion. The general solution in terms of stress intensity factors were obtained by using the Hankel transform and Fourier series. Results of the study for mixed boundary value problem have been represented with the aid of a Fredholm integral equation of the second kind. Also it was concluded that the solutions of a concentric penny-shaped crack in an infinite long orthotropic cylinder and circular plate of infinite radius may be derived from the general solution obtained in this study. Liang and Zang (1992) considered the problem of a concentric penny-shaped crack of Mode III in a finite cylinder. Solution of the problem was obtained by using the Hankel transform and the Fourier series. Results were obtained in terms of stress intensity factors. Furthermore, it was proven that the concentric penny-shaped cracks in an infinite cylinder and infinite circular plate are special cases of the problem of a concentric penny-shaped crack in a finite cylinder. Meshii and Watanabe (2001) studied the development of a practical method to calculate the Mode I stress intensity factor for an inner surface circumferential crack in a finite length cylinder. Thin shell theory formed the bases underlying the developed method in this study. The proposed method has been valid for relatively short cracks and for a wide range of mean radius to wall thickness ratio. Wu and Dzenis (2002) obtained a closed-form solution for the problem of a Mode III edge crack between two bonded elastic strips. The stress intensity factors for the edge crack have been calculated. It was observed that, for the limiting particular cases, the obtained results coincide with the results available in the literature. Lee (2002) considered the problem of stress distribution in a circular cylinder with a circumferential edge crack subjected to uniform shearing stresses. The crack was located on a plane perpendicular to the axis of cylinder and the lateral surface of the cylinder is free of stress. The problem was reduced to the solution of a couple of singular integral equations by using a suitable stress function. These singular integral equations were solved numerically and the stress intensity factors were obtained. Freese and Baratta (2006) obtained solutions for some linear elastic single edge-crack configurations in terms of stress intensity factors. Solutions for various loading conditions have been extracted from the solution of uniformly loaded single edge cracked finite strip configurations. Results for the asymptotic behavior and a common expression for the full range of crack length to strip width ratio has been presented. Yan (2007) considered the problem of a rectangular tensile plate containing an edge crack. A boundary element method proposed by the author has been used to present the stress intensity factors for the considered problem. Furthermore, stress intensity factors of a crack emanating from an edge half-circular hole were calculated. Results obtained in terms of stress intensity factors for two cases have been discussed and it was found that the boundary element method used for the solution was accurate for obtaining the stress intensity factors of crack problems in finite plates. Kaman and Geçit (2008) considered the problem of an axisymmetric finite cylinder of linearly elastic and isotropic material containing a penny-shaped transverse crack. Solution of the complex problem was obtained by the superposition of simpler problems. Moreover, the problem has been reduced to a system of singular integral equations. Then, Gauss–Lobatto and Gauss–Jacobi integration formulas have been used to convert these integral equations to a system of linear algebraic equations. The system of linear algebraic equations has been solved numerically and the results were presented in terms of stress intensity factors at the edges of the rigid support and the crack.

From the literature review, it is observed that the problem of the finite cylinder containing an edge crack has not been solved by the method used in this research study. In this study, an axisymmetric finite cylinder of length $2L$ with clamped ends containing an edge crack subjected to tensile axial loads of uniform intensity $p0$ through the clamping rigid plates at both ends is considered (Figure. 1). The material of the cylinder is assumed to be linearly elastic and isotropic. The lateral surface of the cylinder is free of stresses. The finite cylinder problem is obtained by considering an infinite cylinder containing a ring-shaped crack at $z = 0$ and two rigid penny-shaped inclusions at $z = \pm L$, subjected to tensile axial loads of uniform intensity $p0$ at infinity, and then allowing the radius of the inclusions to approach the radius of the cylinder. The portion of the infinite cylinder between the rigid inclusions then becomes a finite cylinder with clamped ends.

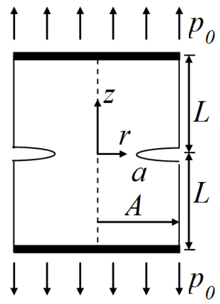

Figure 1. Finite cylinder containing a circumferential edge crack.

2. SOLUTION METHODOLOGY AND DEVELOPMENT OF THE GENERAL EXPRESSIONS

Formulation of the finite cylinder problem is obtained by a procedure starting with considering an infinite cylinder, containing a ring-shaped crack located at $z = 0$ plane and two rigid penny-shaped inclusions located at $z = \pm L$ planes, subjected to tensile axial loads of uniform intensity at infinity, and then letting the radius of the inclusions approach the radius of the cylinder. Solution for the infinite cylinder loaded at infinity having a ring-shaped crack and two penny-shaped rigid inclusions is obtained by superposition of the two simpler problems. Details of the solution procedure may be found in the relevant studies (Durucan, 2010). The linear algebraic equations for an infinite cylinder, containing a ring-shaped crack located at $z = 0$ plane and two rigid penny-shaped inclusions located at $z = \pm L$ planes and subjected to tensile axial loads of uniform intensity at infinity are given below.

$$\sum_{i=1}^{n} C_i \bar{g}_1(\phi_i)\left[\bar{m}_1(\psi_j,\phi_i)+\bar{N}_{11}(\psi_j,\phi_i)\right]+2\sum_{i=1}^{n/2} C_i \bar{g}_2(\eta_i)\eta_i\left[\bar{T}_1(\psi_j,\eta_i)+\bar{N}_{12}(\psi_j,\eta_i)\right]+$$
$$2\sum_{i=1}^{n/2} C_i \bar{g}_3(\eta_i)\eta_i\left[\bar{T}_2(\psi_j,\eta_i)+\bar{N}_{13}(\psi_j,\eta_i)\right]=-(1-\nu), \qquad (\text{j}=1,....,\text{n}-1)$$

$$\sum_{i=1}^{n} C_i \bar{g}_1(\phi_i)\left[\bar{T}_3(\varepsilon_j,\phi_i)+\bar{N}_{21}(\varepsilon_j,\phi_i)\right]+2\sum_{i=1}^{n/2} C_i \bar{g}_2(\eta_i)\left[\eta_i\bar{T}_4(\varepsilon_j,\eta_i)+\bar{m}_2(\varepsilon_j,\eta_i)+\eta_i\bar{N}_{22}(\varepsilon_j,\eta_i)\right]+$$
$$2\sum_{i=1}^{n/2} C_i \bar{g}_3(\eta_i)\eta_i\left[\bar{T}_5(\varepsilon_j,\eta_i)+\bar{N}_{23}(\varepsilon_j,\eta_i)\right]=-\frac{2(1-\nu)\nu}{(\nu+1)(3-4\nu)}, \qquad (\text{j}=1,....,\text{n}/2)$$
$$\sum_{i=1}^{n} C_i \bar{g}_1(\phi_i)\left[\bar{T}_6(\varepsilon_j,\phi_i)+\bar{N}_{31}(\varepsilon_j,\phi_i)\right]+2\sum_{i=1}^{n/2} C_i \bar{g}_2(\eta_i)\eta_i\left[\bar{T}_7(\varepsilon_j,\eta_i)+\bar{N}_{32}(\varepsilon_j,\eta_i)\right]+$$
$$2\sum_{i=1}^{n/2} C_i \bar{g}_3(\eta_i)\left[\eta_i\bar{T}_8(\varepsilon_j,\eta_i)+\bar{m}_3(\varepsilon_j,\eta_i)+\eta_i\bar{N}_{33}(\varepsilon_j,\eta_i)\right]=0, \qquad (\text{j}=1,....,\text{n}/2-1) \qquad \text{(1a-c)}$$

$$\sum_{i=1}^{n} C_i \bar{g}_1(\phi_i)=0,$$

$$\sum_{i=1}^{n/2} C_i \bar{g}_3(\eta_i)\eta_i=0, \qquad \text{(2a,b)}$$

where

$$\phi_i,\eta_i=\cos[(i-1)\pi/(n-1)], \qquad (\text{i}=1,.....,n)$$

$$\psi_j,\varepsilon_j=\cos[(2j-1)\pi/(2n-2)], \qquad (\text{j}=1,.....,n-1)$$

$$C_1=C_n=1/(2n-2), C_i=1/(n-1), \quad (\text{i}=2,.....,\text{n}-1) \qquad \text{(3a-c)}$$

The derivation of the system of linear algebraic equations are given in Durucan (2010).

3. INTEGRAL EQUATIONS

3.1. Finite Cylinder Containing a Ring-Shaped Internal Crack

When the rigid inclusions at $z = \pm L$ spread out ($c \rightarrow A$), the portion of the infinite cylinder between $z = -L$ and $z = L$ becomes a finite cylinder with rigid ends (Fig. 2). In this case, Eqs. (1a-c) are replaced by (see Durucan, 2010 for details)

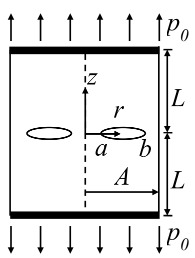

Figure 2. Finite cylinder with a ring-shaped internal crack.

$$\sum_{i=1}^{n} C_i \bar{g}_1(\phi_i)\left[\bar{m}_1(\psi_j,\phi_i)+\bar{N}_{11}(\psi_j,\phi_i)\right]+\frac{2}{\pi}\sum_{i=1}^{n/2} W_i \bar{g}_2(\eta_i)\eta_i\left[\bar{T}_1(\psi_j,\eta_i)+\bar{N}_{12}(\psi_j,\eta_i)\right]+$$
$$\frac{2}{\pi}\sum_{i=1}^{n/2} W_i \bar{g}_3(\eta_i)\eta_i\left[\bar{T}_2(\psi_j,\eta_i)+\bar{N}_{13}(\psi_j,\eta_i)\right]=-(1-\nu), \qquad (\mathrm{j}=1,....,\mathrm{n}-1)$$

$$\sum_{i=1}^{n} C_i \bar{g}_1(\phi_i)\left[\bar{T}_3(\varepsilon_j,\phi_i)+\bar{N}_{21}(\varepsilon_j,\phi_i)\right]+\frac{2}{\pi}\sum_{i=1}^{n/2} W_i \bar{g}_2(\eta_i)\left[\eta_i\bar{T}_4(\varepsilon_j,\eta_i)+\bar{m}_2(\varepsilon_j,\eta_i)+\eta_i\bar{N}_{22}(\varepsilon_j,\eta_i)\right]+$$
$$\frac{2}{\pi}\sum_{i=1}^{n/2} W_i \bar{g}_3(\eta_i)\eta_i\left[\bar{T}_5(\varepsilon_j,\eta_i)+\bar{N}_{23}(\varepsilon_j,\eta_i)\right]=-\frac{2(1-\nu)\nu}{(\nu+1)(3-4\nu)}, \qquad (\mathrm{j}=1,....,\mathrm{n}/2)$$

$$\sum_{i=1}^{n} C_i \bar{g}_1(\phi_i)\left[\bar{T}_6(\varepsilon_j,\phi_i)+\bar{N}_{31}(\varepsilon_j,\phi_i)\right]+\frac{2}{\pi}\sum_{i=1}^{n/2} W_i \bar{g}_2(\eta_i)\eta_i\left[\bar{T}_7(\varepsilon_j,\eta_i)+\bar{N}_{32}(\varepsilon_j,\eta_i)\right]+$$
$$\frac{2}{\pi}\sum_{i=1}^{n/2} W_i \bar{g}_3(\eta_i)\left[\eta_i\bar{T}_8(\varepsilon_j,\eta_i)+\bar{m}_3(\varepsilon_j,\eta_i)+\eta_i\bar{N}_{33}(\varepsilon_j,\eta_i)\right]=0, \qquad (\mathrm{j}=1,....,\mathrm{n}/2-1)$$

(4a-c)

$$\sum_{i=1}^{n} C_i \bar{g}_1(\phi_i)=0$$

$$\sum_{i=1}^{n/2} W_i \bar{g}_3(\eta_i)\eta_i=0$$

(5a,b)

where (Erdogan et al., 1973)

$$P_n^{(-\gamma,-\gamma)}(\eta_i)=0 \qquad (\mathrm{i}=1,.....,n)$$

$$P_{n-1}^{(1-\gamma,1-\gamma)}(\varepsilon_j)=0 \qquad (\mathrm{j}=1,.....,n-1)$$

$$W_i=\frac{2(n-\gamma+1)}{(n+1)!}\frac{\left[\Gamma(n-\gamma+1)\right]^2}{\Gamma(n-2\gamma+1)}\frac{(n-2\gamma+1)^{-1}2^{-2\gamma}}{P_n^{(-\gamma,-\gamma)}(\eta_i)P_{n+1}^{(-\gamma,-\gamma)}(\eta_i)} \qquad (\mathrm{i}=1,.....,n) \qquad (6a\text{-}c)$$

and γ is calculated.

3.2 Finite Cylinder Containing an Edge Crack

In this case, the finite cylinder of radius A and length $2L$ containing a transverse edge crack of width $(A\text{-}a)$ at $z = 0$ subjected to axial tensile loads of uniform intensity $p0$ through the rigid clamps at the ends $z = \pm L$ is considered (Figure 1). In this case, Equations (1a-c) are replaced by (see Durucan, 2010 for details)

$$\sum_{i=1}^{(N-1)/2} C_i \bar{g}_1(\phi_i)\left[\bar{m}_1(\psi_j,\phi_i)+\bar{N}_{11}(\psi_j,\phi_i)\right]+\frac{2}{\pi}\sum_{i=1}^{n/2} W_i \bar{g}_2(\eta_i)\eta_i\left[\bar{T}_1(\psi_j,\eta_i)+\bar{N}_{12}(\psi_j,\eta_i)\right]+$$
$$\frac{2}{\pi}\sum_{i=1}^{n/2} C_i \bar{g}_3(\eta_i)\eta_i\left[\bar{T}_2(\psi_j,\eta_i)+\bar{N}_{13}(\psi_j,\eta_i)\right]=-(1-\nu), \qquad (\mathrm{j}=1,....,(\mathrm{N}-1)/2)$$

$$\sum_{i=1}^{(N-1)/2} C_i \bar{g}_1(\phi_i)\left[\bar{T}_3(\varepsilon_j,\phi_i)+\bar{N}_{21}(\varepsilon_j,\phi_i)\right]+\frac{2}{\pi}\sum_{i=1}^{n/2} W_i \bar{g}_2(\eta_i)\left[\eta_i\bar{T}_4(\varepsilon_j,\eta_i)+\bar{m}_2(\varepsilon_j,\eta_i)+\eta_i\bar{N}_{22}(\varepsilon_j,\eta_i)\right]+$$
$$\frac{2}{\pi}\sum_{i=1}^{n/2} W_i \bar{g}_3(\eta_i)\eta_i\left[\bar{T}_5(\varepsilon_j,\eta_i)+\bar{N}_{23}(\varepsilon_j,\eta_i)\right]=-\frac{2(1-\nu)\nu}{(\nu+1)(3-4\nu)}, \qquad (\mathrm{j}=1,....,\mathrm{n}/2)$$

$$\sum_{i=1}^{(N-1)/2} C_i \bar{g}_1(\phi_i)\left[\bar{T}_6(\varepsilon_j,\phi_i)+\bar{N}_{31}(\varepsilon_j,\phi_i)\right]+\frac{2}{\pi}\sum_{i=1}^{n/2} W_i \bar{g}_2(\eta_i)\eta_i\left[\bar{T}_7(\varepsilon_j,\eta_i)+\bar{N}_{32}(\varepsilon_j,\eta_i)\right]+$$
$$\frac{2}{\pi}\sum_{i=1}^{n/2} W_i \bar{g}_3(\eta_i)\left[\eta_i \bar{T}_8(\varepsilon_j,\eta_i)+\bar{m}_3(\varepsilon_j,\eta_i)+\eta_i \bar{N}_{33}(\varepsilon_j,\eta_i)\right]=0, \qquad (j=1,\ldots, n/2-1) \tag{7a-c}$$

$$\sum_{i=1}^{n/2} W_i \bar{g}_3(\eta_i)\eta_i = 0 \tag{8}$$

N is selected to be an odd number, η_i $(i=1,\ldots,n)$ and ε_j $(j=1,\ldots,n-1)$ are calculated again from Eqs. (6a,b) and

$$\psi_j = \cos[(2j-1)\pi/(2N-2)], \qquad (j=1,\ldots,N-1)$$

$$\phi_i = \cos[(i-1)\pi/(N-1)], \qquad (i=1,\ldots,N) \tag{9a,b}$$

3.3. Stress Intensity Factors

Stress intensity factors form a very important basis in fracture mechanics. Stresses become infinity in the vicinity of tips or edges of cracks and inclusions. These infinite stresses are represented by means of stress intensity factors. Mode I stress intensity factors at the edges of the internal ring-shaped crack are defined and calculated in the form

$$k_{1a} = \lim_{r\to a}\sqrt{2(a-r)}\,\sigma_z(r,0) = \frac{\sqrt{2}\mu}{(1-\nu)}\frac{g_1^*(a)}{\sqrt{b-a}}$$

$$k_{1b} = \lim_{r\to b}\sqrt{2(r-b)}\,\sigma_z(r,0) = -\frac{\sqrt{2}\mu}{(1-\nu)}\frac{g_1^*(b)}{\sqrt{b-a}} \tag{10a,b}$$

Mode I stress intensity factor at the root of the edge crack may be defined and calculated as

$$k_{1a} = \lim_{r\to a}\sqrt{2(a-r)}\,\sigma_z(r,0) = \frac{\sqrt{2}\mu}{(1-\nu)} g_1^*(a) \tag{11}$$

Mode I and II stress intensity factors at the corners of the finite cylinder with stiffened ends are

$$k_{1A} = \lim_{r\to A}\sqrt{2}(A-r)^{\gamma}\,\sigma_z(r,L)$$

$$k_{2A} = \lim_{r\to A}\sqrt{2}(A-r)^{\gamma}\,\tau_{rz}(r,L) \tag{12a,b}$$

$$k_{1A} = \frac{\sqrt{2}}{2}\left\{\frac{1}{2(1-\nu)}\frac{p_1^*(A)}{(2A)^{\gamma}\sin\pi\gamma}\left[(2\nu-1)(\cos\pi\gamma+1)+4(1-\nu)(\gamma-1)-2(\gamma-1)^2\right]-\frac{p_2^*(A)}{(2A)^{\gamma}}\right\}$$

$$k_{2A} = \frac{\sqrt{2}}{2}\left\{\frac{1}{2(1-\nu)}\frac{p_2^*(A)}{(2A)^{\gamma}\sin\pi\gamma}\left[(1-2\nu)(\cos\pi\gamma+1)+4(1-\nu)(\gamma-1)+2(\gamma-1)^2\right]-\frac{p_1^*(A)}{(2A)^{\gamma}}\right\} \tag{13a,b}$$

4. NUMERICAL RESULTS

The system of linear algebraic equations, Eqs. (4a-c) and (5a,b), or Eqs. (7a-c) and (8), are solved numerically and the unknown functions g_1 (ϕ), g_i (η) (i = 2,3) are calculated at discrete collocation points. Then, the physically significant quantities, for example, the stress intensity factors at the edges of the crack, at the edges of the inclusion and around the clamped corners of the finite cylinder can be calculated.

4.1. Finite Cylinder with Clamped Ends Containing an Internal Ring-Shaped Crack

Normalized stress intensity factors may be defined and calculated as:

$$\bar{k}_{1a} = \frac{k_{1a}}{p_0\sqrt{(b-a)/2}} = \frac{\bar{g}_1(-1)}{4(1-\nu)},$$

$$\bar{k}_{1b} = \frac{k_{1b}}{p_0\sqrt{(b-a)/2}} = -\frac{\bar{g}_1(1)}{4(1-\nu)}, \qquad \text{(14a,b)}$$

for the internal crack.

When the radius of the rigid inclusions approaches the outer radius of the cylinder ($c \to A$), the portion of the infinite cylinder becomes identical with a finite cylinder with clamped ends whose outer sides are under the action of uniformly distributed normal forces and an internal ring-shaped crack. Figure 3 shows variations of the normalized Mode I stress intensity factors $\bar{k}_{1a}$ and $\bar{k}_{1b}$ defined in Eqs. (14a,b) at the edges of the crack with relative crack width (b-a)/A when b+a = A and ν = 0.3. Results are given for L/A = 0.25 and 1 noting that the numerical results for $\bar{k}_{1a}$ and $\bar{k}_{1b}$ remain unchanged for values of L/A greater than 1. Figure 4 shows variations of the normalized stress intensity factors $\bar{k}_{1A}$ and $\bar{k}_{2A}$ defined as

$$\bar{k}_{iA} = \frac{k_{iA}}{p_0 A^{\gamma}} \qquad (\mathrm{i} = 1,.2) \qquad \text{(15a,b)}$$

$$\bar{k}_{1A} = \frac{\sqrt{2}}{2}\left\{\frac{1}{2(1-\nu)}\frac{\bar{g}_2(1)}{2^{\gamma}\sin\pi\gamma}\left[(2\nu-1)(\cos\pi\gamma+1)+4(1-\nu)(\gamma-1)-2(\gamma-1)^2\right]-\frac{\bar{g}_3(1)}{2^{\gamma}}\right\}$$

$$\bar{k}_{2A} = \frac{\sqrt{2}}{2}\left\{\frac{1}{2(1-\nu)}\frac{\bar{g}_3(1)}{2^{\gamma}\sin\pi\gamma}\left[(1-2\nu)(\cos\pi\gamma+1)+4(1-\nu)(\gamma-1)+2(\gamma-1)^2\right]-\frac{\bar{g}_2(1)}{2^{\gamma}}\right\} \qquad \text{(16a,b)}$$

where γ is calculated with (b-a)/A when b+a = A and ν = 0.3. It can be observed from this figure that, $\bar{k}_{1A}$ is much larger than $\bar{k}_{2A}$. For relatively small aspect ratio L/A, $\bar{k}_{1A}$ and $\bar{k}_{2A}$ increase as the outer edge of the crack gets closer to the lateral surface of the cylinder which means also that it is getting closer to the corners at which the stress intensity factors are calculated. This is expected, since the edge of the crack increases the prevailing stress levels. However, for relatively large values of L/A, $\bar{k}_{1A}$ and $\bar{k}_{2A}$ appear to be insensitive to crack width.

4.2. Finite Cylinder with Clamped Ends Containing a Circumferential Edge Crack

When the outer radius of the internal ring-shaped crack approaches the outer radius of the cylinder ($b \to A$), the case of a finite cylinder with a circumferential edge crack is obtained. Figure 5 shows the normalized Mode I stress intensity factor

$$\bar{k}_{1a} = \frac{k_{1a}}{p_0\sqrt{2(A-a)}} = \frac{\bar{g}_1(-1)}{4(1-\nu)} \qquad \text{(17)}$$

at the inner edge of the circumferential edge crack. Figure 5 shows the variation of $\bar{k}_{1a}$ with the relative crack depth (A-a)/A when ν = 0.3 for L/A = 0.25 and for $L/A \geq 1$. $\bar{k}_{1a}$ increases with increasing crack depth. It is smaller for shorter cylinders when the crack is relatively narrow and larger when the crack is relatively deep. Figure 6 shows the variation of $\bar{k}_{1A}$ and $\bar{k}_{2A}$ defined in Eqs. (16a,b) with the relative crack depth (A-a)/A when L/A = 1 for ν = 0.1, 0.3 and ~ 0.5. It may be observed from this figure that, increase in the value of ν increases $\bar{k}_{2A}$, but decreases $\bar{k}_{1A}$. Both $\bar{k}_{1A}$ and $\bar{k}_{2A}$ decrease slightly, in general, with increasing crack depth. $\bar{k}_{1A}$ is always larger than $\bar{k}_{2A}$.

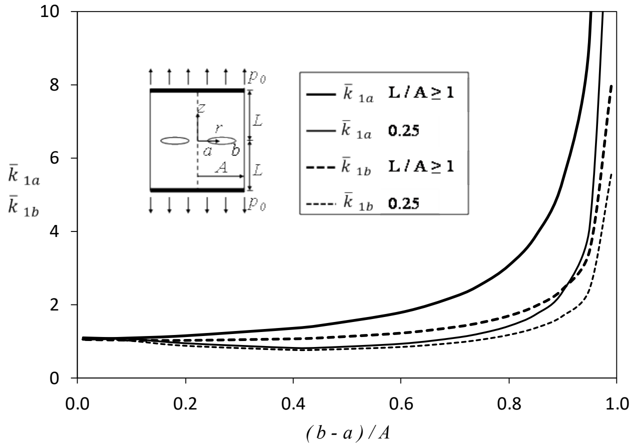

Figure 3. Normalized stress intensity factors $\bar{k}_{1a}$ and $\bar{k}_{1b}$ at the edges of the internal crack when ν = 0.3 and b+a = A in finite cylinder.

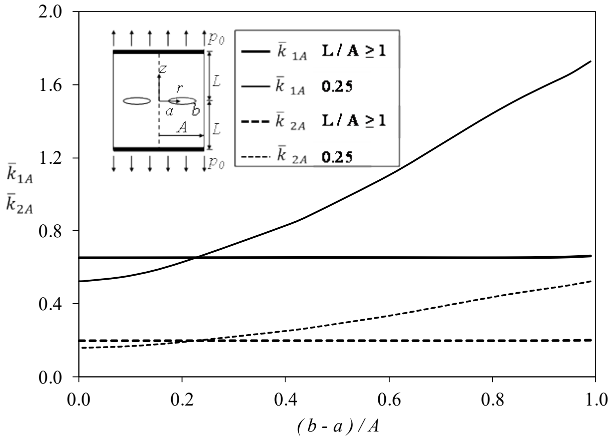

Figure 4. Normalized stress intensity factors $\bar{k}_{1A}$ and $\bar{k}_{2A}$ at the corner of the finite cylinder with a ring-shaped internal crack when ν = 0.3 and b+a = A.

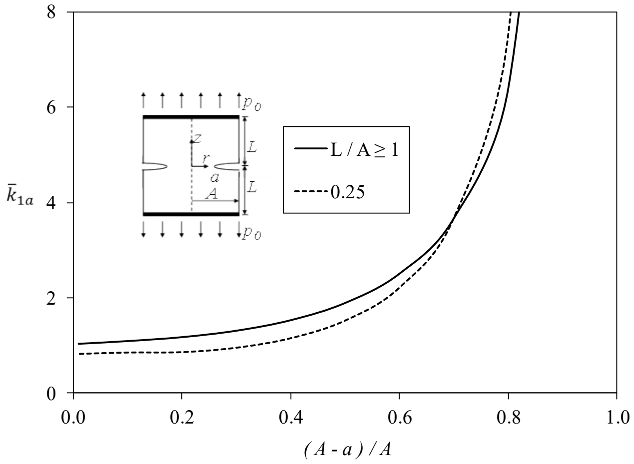

Figure 5. Normalized stress intensity factor $\bar{k}_{1a}$ at the inner edge of the edge crack in finite cylinder when ν = 0.3.

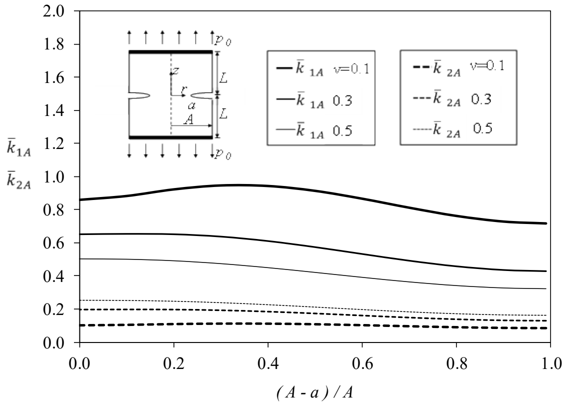

Figure 6. Normalized stress intensity factors $\bar{k}_{1A}$ and $\bar{k}_{2A}$ at the corner of the finite cylinder containing an edge crack when $L = A$.

5. ACKNOWLEDGEMENT

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Nomenclature

a, b	Inner and outer radii of ring-shaped crack
A	Radius of cylinder
c	Radius of penny-shaped inclusions
C_i	Weighting constants of the Gauss-Lobatto polynomials
d_{ij}	Coefficient functions
$g_1(r)$	Crack surface displacement derivative
$g_1^*(r)$	Hölder-continuous function on crack
$\overline{g_1}(r)$	Normalized bounded part of $g_1(r)$
$g_2(r)$, $g_3(r)$	Normal and shear stress jumps on rigid inclusions
$g_2^*(r)$, $g_3^*(r)$	Hölder-continuous functions on inclusions
$\overline{g_2}(r), \overline{g_3}(r)$	Normalized bounded parts of $g_2(r)$, $g_3(r)$
I_0, K_0, I_1, K_1	Modified Bessel functions of the 1st and 2nd kinds of order zero and one
J_0, J_1	Bessel functions of the 1st kind of order zero and one
k_{1a}, k_{1b}	Mode I stress intensity factors at the edges of crack
k_{1c}, k_{2c}	Stress intensity factors at the edge of internal rigid inclusions
$\bar{k}_{1a}, \bar{k}_{2b}$	Normalized stress intensity factors at the edges of crack
$\bar{k}_{1c}, \bar{k}_{2c}$	Normalized stress intensity factors at the edge of internal rigid inclusions
K, E	Complete elliptic integrals of the 1st and the 2nd kinds
L_{ij}	Integrands of the kernels N_{ij}
$L_{ij\infty}$	Dominants part of L_{ij} as $\alpha\to\infty$
L	Distance between crack and inclusions
m_i, M_i, N_{ij}, T_i	Kernels of the integral equations
N_{ijb}, N_{ijs}	Bounded and singular parts of N_{ij} as $\alpha\to\infty$
p_0	Intensity of the axial tensile load

$P_n(\alpha,\beta)$	Jacobi polynomials
r, z	Cylindrical coordinates
t	Integration variable
u, w	Displacement components in r- and z-directions
W_i	Weighting constants of the Jacobi polynomials
α	Fourier transform variable
β, θ, γ	Powers of singularity at the edges of the crack an inclusion
η, ε	Normalized variables on inclusions
ϕ, ψ	Normalized variables on crack
μ	Shear modulus of elasticity
ν	Poisson's ratio
σ, τ	Normal and shear stresses
σ_{zb}, σ_{zs}	Bounded and singular parts of σ_z at the edges of the crack and inclusions