Effects Of Magnetic Field, Buoyancy And Conjugate Heat Transfer On Flow Over A Vertical Thin Plate

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ABSTRACT: In this study, effects of magnetic field, conjugate heat transfer and buoyancy have been numerically investigated on flow over a thin vertical plate. The fluid is assumed to be incompressible and dense. The nonlinear coupled parabolic partial differential equations governing the flow are transformed into the boundary layer equations, which are then solved numerically using the Keller box method. The effects of the conjugate heat transfer parameter p, the mixed convection parameter Ri, the magnetic parameter Mn and the electric field parameter E1 on the velocity and temperature profiles as well as on the local skin friction and local heat transfer are presented and analyzed. The validity of the methodology and analysis is checked by comparing the results obtained for some specific cases with those available in the literature.

Keywords: *Conjugate Heat Transfer, MHD Flow, Mixed Convection, Boundary Layer*

Düşey İnce Bir Plaka Üzerinden Olan Akışa Manyetik Alan, Kaldırma Kuvveti Ve Birleşik Isı Transferinin Etkisi

ÖZET: Bu çalışmada, dikey ince bir levha üzerinden akışa manyetik alan, birleşik ısı transferi ve kaldırma kuvveti etkileri numerik olarak araştırılmıştır. Akışkanın sıkıştırılamaz ve yoğun olduğu varsayılmıştır. Akışı karakterize eden, lineer olmayan bağımlı parabolik kısmi diferansiyel denklemler sınır tabaka denklemlerine dönüştürülmüş, sonrasında ise Keller-box metodu kullanılarak numerik çözülmüştür. Birleşik ısı transferi parametresi p, kaldırma kuvveti etkisi Ri, manyetik alan etkisi Mn ve elektrik alan etkisinin E1 sınır tabaka içindeki hız ve sıcaklık ile local sürtünme ve local ısı transferi parametrelerine etkisi gösterilmiştir. Kullanılan yöntemin doğruluğu literatürde bazı özel durumlar için elde edilen sonuçlarla karşılaştırılmıştır.

Anahtar Kelimeler: *Birleşik Isı Transferi, MHD Akışı, Birleşik Taşınım, Sınır Tabaka*

1. INTRODUCTION

The interaction between the conduction inside and the buoyancy forced flow of fluid along a solid surface is termed as conjugate heat transfer process [1]. In many practical applications, such as heat exchangers, heaters, nuclear reactors, pipe insulation systems, etc., the effect of conduction within the solid wall is significant and must be taken into account. Hence, the analysis of this type of heat transfer mechanisms possesses necessary the coupling of the conduction in the solid body and the convection in the fluid surrounding it [2].

The conjugate heat transfer problem, in which the coupled heat transfer processes between the solid body (conduction mechanisms) and the fluid flow (convection mechanisms) are considered simultaneously, has been investigated by several researchers. Mamun et al. [1] investigated the heat generation effect on natural convection flow along and conduction inside a vertical flat plate. Chang [2] presented a numerical analysis of the flow and heat transfer characteristics of mixed convection in a micropolar fluid flowing along a vertical flat plate with conduction effects. He assumed that the heat conduction in the wall was only in the transversal direction. Miyamoto et al. [3] studied two-dimensional conjugate heat transfer problems of free convection from a vertical flat plate with a uniform temperature or a uniform heat flux at the outsidem surface of the plate. Sparrow and Chyu [4] investigated the conjugate problem for a vertical plate fin with various heat transfer coefficients under forced convection. They assumed that the heat conduction in the fin was to be one-dimensional. Char et al. [5] employed the cubic spline collocation numerical method to analyze the conjugate heat transfer in the laminar boundary layer on a continuous, moving plate. Wang [6] studied the thermo-fluid-dynamic field resulting from the coupling of wall conduction with laminar mixed convection heat transfer of micropolar fluids along a vertical flat plate. Pop et al. [7] presented a detailed numerical study of the

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conjugate mixed convection flow along a vertical flat plate. Hsiao and Hsu [8] studied a conjugate mixed convection heat transfer problem of a second-grade viscoelastic fluid past a horizontal flat-plate fin.

This article illustrates the effects of wall conduction on MHD mixed convection flow over a thin vertical plate. The boundary layer equations governing the flow are reduced to local non-similarity equations which are solved using the implicit finite difference method (Keller box). Variation in the fluid–solid interfacial temperature distribution, the local skin friction and local heat transfer parameters as well as the velocity and temperature profiles are presented to highlight the influence of the wall conduction, buoyancy, electric field and magnetic effects parameters.

2. ANALYSIS

This study considers two-dimensional, steady, laminar, incompressible electrically conducting fluid flow over a thin vertical plate of length L and finite thickness b (L>>b). The physical model and coordinate system are shown in Figure 1.

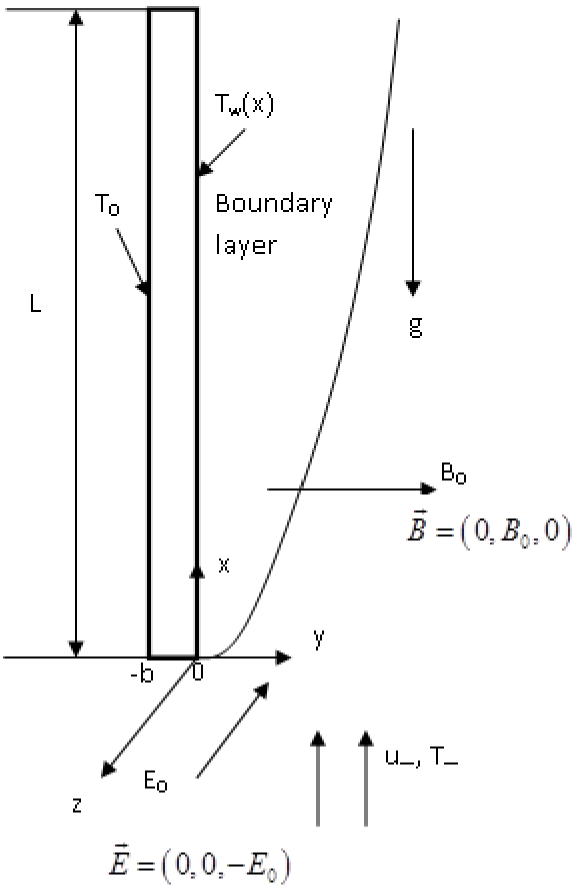


Figure 1. The schematic of the problem.

Far above/below the surfaces of the thin vertical plate, the velocity and the temperature of the free stream are u_∞ and T_∞, respectively. The temperature of the inside surface of the plate is maintained at a constant temperature of T_0, where$T_0 > T_\infty$. The flow region is exposed under uniform transverse magnetic fields $\vec{B}=(0,B_0,0)$ and uniform electric field $\vec{E}=(0,0,-E_0)$ [9]. Magnetic and electric fields are known from Maxwell's equation that $\nabla.\vec{B}=0$ and $\nabla x\vec{E}=0$. When magnetic field is not so strong then electric field and magnetic field obey Ohm's law $\vec{J}=\sigma\left(\vec{E}+\vec{q}\,x\,\vec{B}\right)$, where $\vec{J}$ is the Joule current, σ is the magnetic permeability and $\vec{q}$ is the fluid velocity. It is assumed that magnetic Reynolds number of the fluid is small so that induced magnetic field and Hall effect may be neglected and take into account of magnetic field effect as well as electric field in momentum boundary layer equation [9]. Under foregoing assumptions and taking into account the Boussinesq approximation and the boundary layer approximation, the system of continuity, momentum and energy equations can be written:

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+\frac{\sigma}{\rho}\left[E_0B_0-B_0^2\left(u-u_\infty\right)\right]+g\,\beta\left(T-T_\infty\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial y^2}\right) \tag{3}$$

The appropriate boundary conditions for the velocity and temperature of this problem are:

$$\begin{array}{llll} x=0 & y>0 & T=T_\infty & u=u_\infty \\ x>0 & y=0 & T=T_w(x) & u=0 \quad \upsilon=0 \\ y\to\infty & & T\to T_\infty & u\to u_\infty \end{array} \tag{4}$$

where subscripts w and ∞ refer to the wall and the boundary layer edge, respectively. In addition, $T_w(x)$ is the surface temperature of the plate, which is not known a priori.

One objective of the current study is to predict the surface temperature of the plate $T_w(x)$. Therefore, an additional governing equation is required for the plate based on the simplification that the plate steadily transfers its heat to the surrounding fluid. Since the thickness of the plate, *b*, is small compared with its length, L, the axial conduction term in the heat conduction equation of the thin plate can be omitted [2]. The governing equation for the temperature distribution within the plate is given by

$$\left.\frac{d^2T}{dy^2}\right|_s = 0; 0 \le x \le L; -b < y \le 0 \qquad (5)$$

The boundary conditions for the wall of plate are given:

At $y = -b, T_s = T_0,$ (6a)

At the interface

$$(y=0),\ T_s = T(x,0):$$
$$-k_s \left.\frac{dT}{dy}\right|_{y=0,s} = -k_f \left.\frac{\partial T(x,0)}{\partial y}\right|_{y=0,f} \qquad (6b)$$

where k_s and k_f are the thermal conductivity of the plate and the fluid, respectively. The boundary conditions given in Eq. (6b) state the physical requirements that the temperature and heat flux of the plate must be continuous across the solid–fluid interface. From Eqs. (5) and (6), the temperature distribution T_w at the interface is shown to be:

$$T_w(x) = T(x,0) = b\frac{k_f}{k_s}\frac{\partial T(x,0)}{\partial y} + T_0 \qquad (7)$$

To seek a solution, the following dimensionless variables are introduced:

$$\xi(x) = \frac{x}{L}, \psi(x,y) = (\nu u_\infty x)^{1/2} f(\xi,\eta),$$
$$\eta = y\left(\frac{u_\infty}{\nu x}\right)^{1/2}, \ \theta = \frac{T - T_\infty}{T_0 - T_\infty} \qquad (8)$$

where $\psi(x,y)$ is the free stream function that satisfies Eq. (1) with $u = \partial\psi/\partial y$ and $\upsilon = -\partial\psi/\partial x$

In terms of these new variables, the velocity components can be expressed as

$$u = u_\infty f', \qquad (9)$$
$$\upsilon = -\frac{(\nu u_\infty x)^{1/2}}{x}\left\{\frac{1}{2}f + \xi\frac{\partial f}{\partial \xi} - \frac{\eta}{2}f'\right\} \qquad (10)$$

The transformed momentum and energy equations together with the boundary conditions, Eqs. (2), (3) and (5), can be written as

$$f''' + \frac{1}{2}ff'' + Mn\xi\left[E_1 - (f'-1)\right] + Ri\xi\theta = \frac{\xi}{\varepsilon^2}\left(f'\frac{\partial f'}{\partial \xi} - f''\frac{\partial f}{\partial \xi}\right) \qquad (11)$$

$$\frac{1}{\Pr}\theta'' + \frac{1}{2}f\theta' = \xi\left(f'\frac{\partial \theta}{\partial \xi} - \theta'\frac{\partial f}{\partial \xi}\right) \qquad (12)$$

with the boundary conditions;

$$f(\xi,0) + 2\xi\frac{\partial f}{\partial \xi} = 0,\ f'(\xi,0) = 0,$$
$$\theta(\xi,0) - 1 = p\xi^{-1/2}\theta'(\xi,0),\ f'(\xi,\infty) = 1,$$
$$\theta(\xi,\infty) = 0 \qquad (13)$$

where $p = \frac{k_f}{k_s}\frac{b}{L}\mathrm{Re}_L^{1/2}$ is the conjugate heat transfer parameter. It should be noticed that for the limiting case of p = 0, the thermal boundary condition in Eq. (13) on the wall becomes isothermal. Hence, the magnitude of p determines the importance of the wall heat conduction effect [2].

The corresponding dimensionless groups that appeared in the governing equations defined as:

$$\Pr = \frac{\mu c_p}{k} = \frac{\nu}{\alpha}, \quad Mn = \frac{Ha}{\mathrm{Re}}, \quad Ha = \frac{\sigma B_0^2 L^2}{\mu},$$
$$E_1 = \frac{E}{\mathrm{Re}}, \ E = \frac{E_0}{B_0 \nu/L},$$
$$Ri = \frac{Gr_L}{\mathrm{Re}_L},\ Gr_L = \frac{g\beta(T_w - T_\infty)KL}{\nu^2},\ \mathrm{Re}_L = \frac{u_\infty L}{\nu} \qquad (14)$$

where Mn is the magnetic parameter; Ha is the Hartman number; E_1 is the electric field parameter; Ri is the Richardson number, Gr is the Grashof number; Re is the Reynolds number; Pr is the Prandtl number.

From the definition of the dimensionless wall temperature, it can be shown that

$$\theta_w = \frac{T_w - T_\infty}{T_0 - T_\infty} \qquad (15)$$

In this study, the Keller's box finite-difference method is used in the solution. The system of transformed equations together with the boundary conditions, Eqs. 11–13, is solved numerically by an efficient and accurate finite-difference scheme similar to that described in Cebeci and Bradshaw [10] and Takhar and Beg [11]. This numerical scheme with its very desirable features is very appropriate for the solution of parabolic partial differential equations. A uniform grid with a step size 0.01 in the η-direction and a non-uniform grid in the ξ -direction are used. Resolution of the grid sizes was kept satisfactory enough in terms of the convergence of the numerical solution and the accuracy of the results.

In order to verify the accuracy of the present method, the present results were compared with those of Lloyd and Sparrow [12] and Chang [2] (Table 2). The comparison is found to be in good agreement, as shown in Table 1.

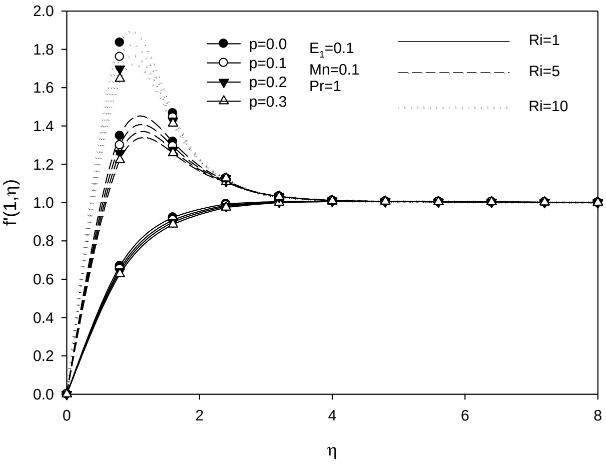
Table 1. Comparison of the values $-\theta'(\xi,0)$ for various values ξ with Pr=10, Mn=0.0, Ri=7.8, E_1=0.0 and p=0.0.

ξ	Lloyd and Sparrow [12]	Chang [2]	Present Study
0.00000	0.7281	0.7280	0.7278
0.00125	0.7313	0.7291	0.7318
0.00500	0.7404	0.7373	0.7403
0.01250	0.7574	0.7566	0.7574
0.05000	0.8259	0.8351	0.8289
0.12500	0.9212	0.9412	0.9397
0.25000	1.0290	1.0603	1.0601

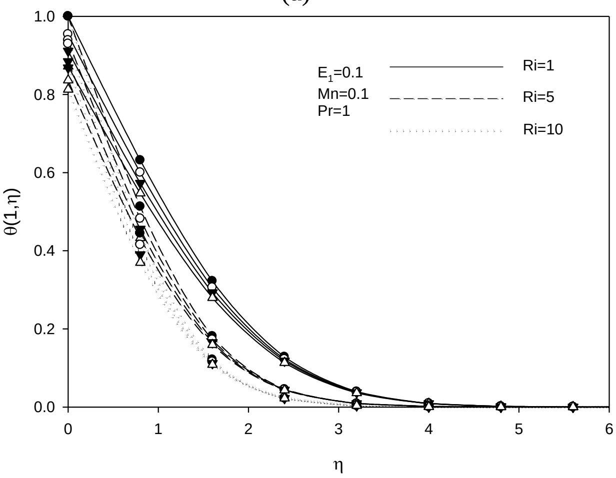
3. RESULTS AND DISCUSSION

The aim of this study was to investigate the flow and heat transfer characteristics for the mhd mixed convection flow over a thin vertical plate with wall conduction effect. The following ranges of the main parameters are considered: conjugate heat transfer parameter p=0.0, 0.1, 0.2 and 0.3; mixed convection parameter Ri =1.0, 5.0 and 10.0; Pr=1.0; magnetic interaction parameter Mn= 0.0, 0.5, 1.0 and 2.0 and electric field parameter E_1=0.1, 0.5, 1.0 and 2.0.

The effect of conjugate heat transfer parameter p on the velocity and temperature profiles within the boundary layer is shown in Fig. 2 (a) and (b), respectively.



(a)

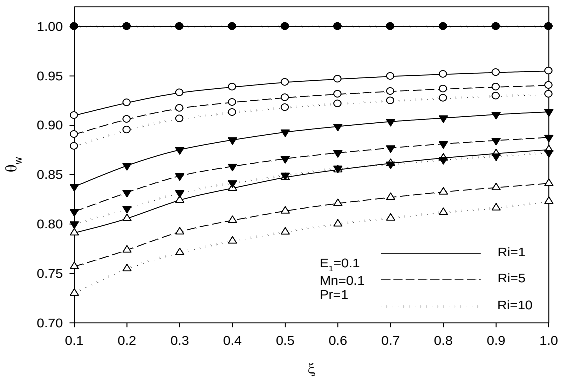


(b)

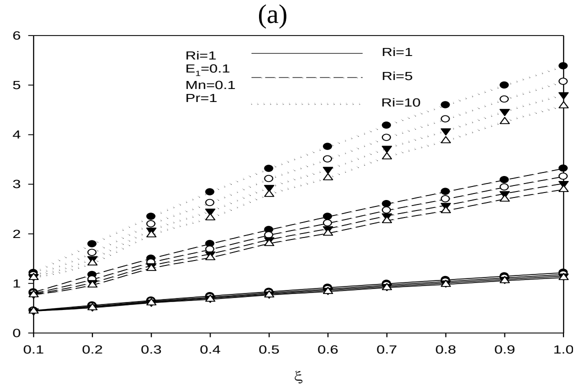
Figure 2. Dimensionless velocity (a) and temperature (b) profiles for different p and Ri while Pr=1.0, Mn=0.1, E_1=0.1 and ξ=1.

The increasing of the conjugate heat transfer parameter decreases velocity and temperature gradients at the wall. A lower wall conductance k_s or higher convective cooling effect due to greater k_f increases the value of p as well as causes greater temperature difference between the two surfaces of the plate [2]. The temperature at the solid-fluid interface is reduced since the temperature at the outside surface of the plate is kept constant. Figure 2 also shows for different values of the buoyancy parameter Ri. The increasing of Ri increases velocity and temperature gradients at the wall.

The variation of the interfacial temperature, the local skin friction and the local heat transfer parameters for different values of p with ξ are shown in Fig. 3(a), (b) and (c), respectively. It can be seen that the temperature of the fluid on the wall increases with ξ for a given value of p (Fig. 3(a)). Comparing with isothermal wall (p = 0), an increase in the conjugate heat transfer parameter, p, causes a reduction in the interfacial temperature [2]. The increases value of p decreases the local skin friction and the local heat transfer parameters as shown in Fig. 3(b) and (c). Also, increasing Ri decreases the interfacial temperature and increases the local skin friction and the local heat transfer parameters (Fig.3 (a), (b) and (c)). Similar results were found in the literature [2].



(a)



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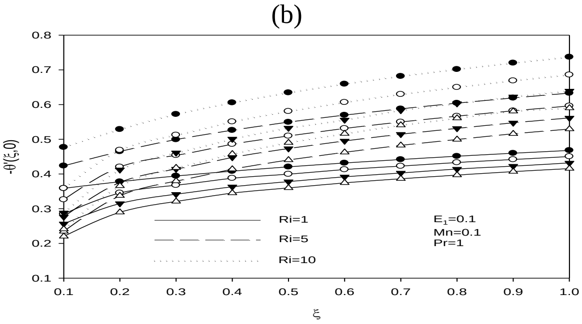
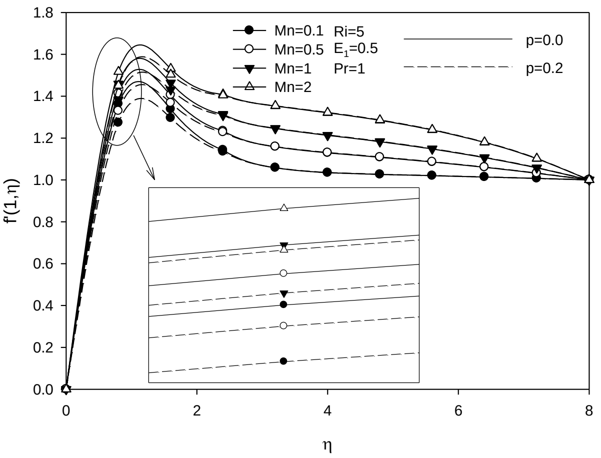


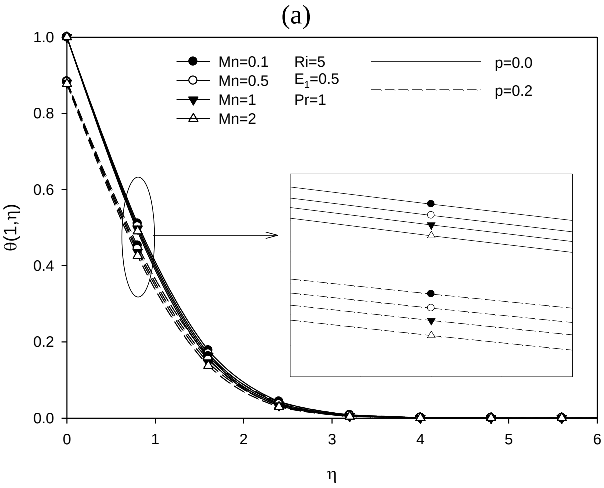
(c)

Figure 3. Effects of p and Ri on the dimensionless interfacial temperature (a), local skin friction (b) and local heat transfer (c) parameters against the streamwise distance ξ while Pr=1.0, Mn=0.1 and E_1=0.1.

Figure 4. shows the dimensionless velocity (a) and temperature (b) profiles inside the boundary layer for different values of the magnetic parameter Mn for the cases of isothermal plate (p=0) and non-isothermal plate (p>0.0). The increasing of the magnetic parameter Mn increases velocity and temperature gradients at the wall due to magnetic field effects on external flow field. As mentioned above, increasing the conjugate heat transfer parameter p decreases velocity and temperature gradients at the wall.



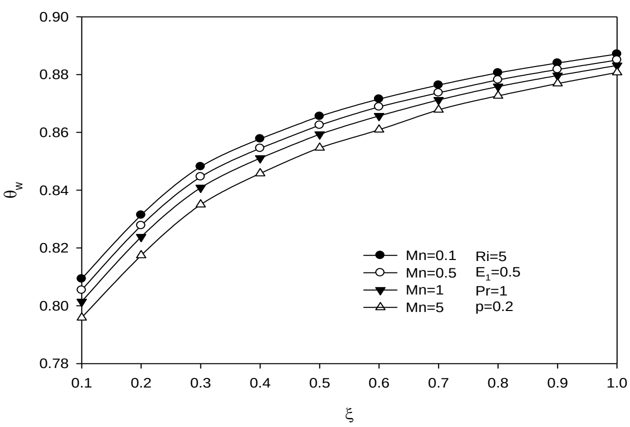
(a)



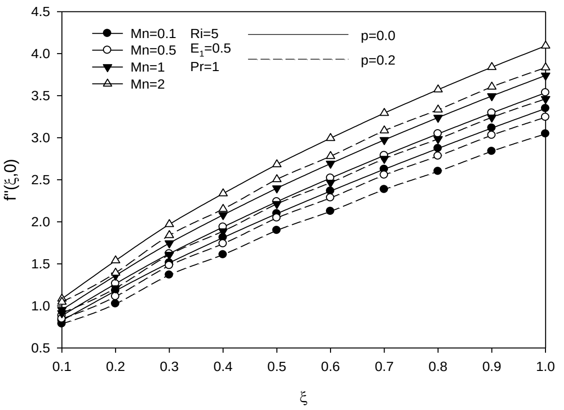
(b)

Figure 4. Dimensionless velocity (a) and temperature (b) profiles for different Mn while Pr=1.0, Ri=5, E1=0.1 and ξ=1.

As seen in Fig.5 increasing the magnetic parameter Mn decreases the interfacial temperature (Fig. (5a)). The magnetic force aiding the flow and increases the local skin friction (i.e. shear stress) and the local heat transfer (i.e. heat transfer rate) parameters at the wall (Fig. 5 (b) and (c)).



(a)



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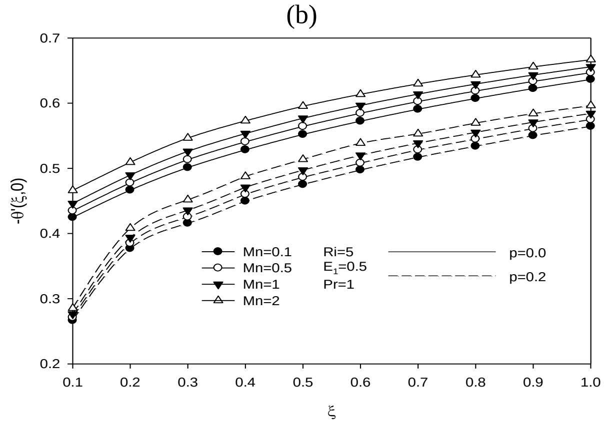
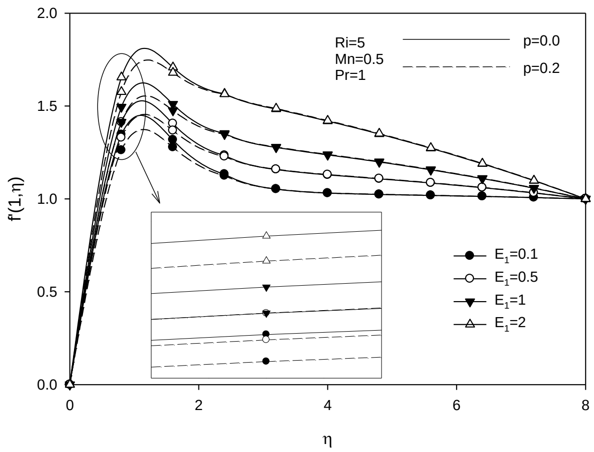


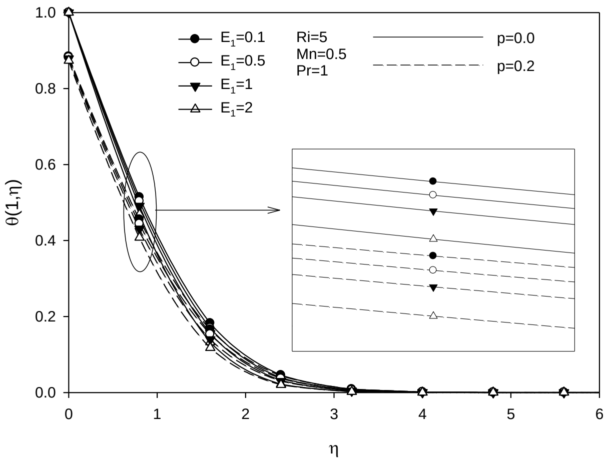
(c)

Figure 5. Effect of Mn on the dimensionless interfacial temperature (a), local skin friction (b) and local heat transfer (c) parameters against the streamwise distance ξ while Pr=1.0, Ri=5 and E1=0.1.

The dimensionless velocity (a) and temperature (b) profiles inside the boundary layer for different values of the electric field parameter E_1 for the cases of isothermal plate (p=0) and non-isothermal plate (p>0.0) are illustrated in Fig 6.



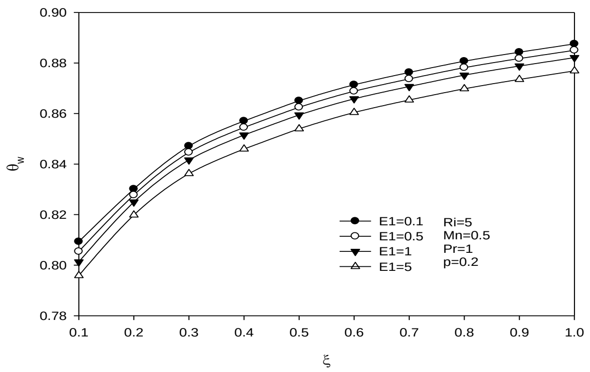
(a)



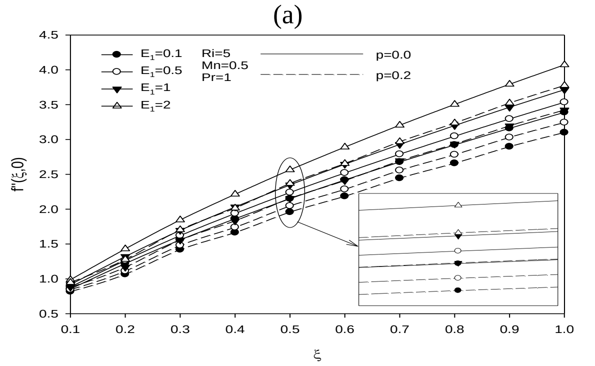
(b)

Figure 6. Dimensionless velocity (a) and temperature (b) profiles for different E1 while Pr=1.0, Ri=5, Mn=0.1 and ξ=1.

Increasing of the electric field parameter E_1 decreases the momentum and thermal boundary layers thickness because Lorentz force arising due to electric field acts as an accelerating force in reducing the frictional resistance [9]. Therefore, increasing the velocity and temperature gradients at wall increases the local skin friction and local heat transfer parameters as shown in Fig. 7 (b) and (c). Also increasing the electric field parameter E_1 decreases the interfacial temperature (Fig. (7a)).



(a)



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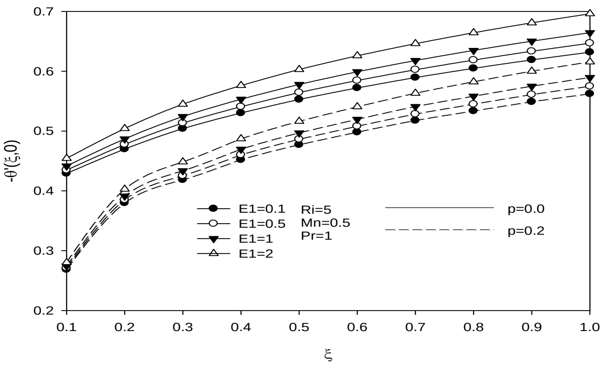

(c)

Figure 7. Effect of E1 on the dimensionless interfacial temperature (a), local skin friction (b) and local heat transfer (c) parameters against the streamwise distance ξ while Pr=1.0, Ri=5 and Mn=0.1.

4. CONCLUSIONS

This study has analyzed the effects of wall conduction and buoyancy on MHD mixed convection flow. The influences of the conjugate heat transfer parameter, mixed convection parameter, magnetic interaction and electric field parameter on the solid–liquid interfacial temperature distribution, the local skin friction and the local heat transfer parameters have been systematically examined. From the present numerical investigation, the following conclusions can be drawn:

An increase in the conjugate heat transfer parameter decreases the velocity and the temperature gradient and therefore decreases the dimensionless interfacial temperature distribution, the local skin friction and the local heat transfer parameters.

An increase in the magnetic, electric field and buoyancy parameters increase the local skin friction and local heat transfer parameters and decrease the dimensionless interfacial temperature distributions. Increasing Mn, E_1 and Ri decreases the velocity and temperature gradients at wall for non-isothermal cylinder (i.e.p>0).

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