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Çevresel Titreşim Kullanılarak Yapı Dinamik Parametrelerinin Optimal Belirlenmesi

Optimal Determination of Structural Dynamical Parameters Using Ambient Vibration

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ABSTRACT

In this study, by using ambient vibration, a new approach based on improvement and correction of system characteristic matrix in modal vibration is provided. The result is that actual system characteristic matrices are accurately made such that the error is minimized at great extent. This clearly shows how the system parameters can be updated in a more reliable way. Firstly, by approximation, the actual system characteristic matrices are determined using the singular value decomposition of block Hankel matrix that is built from response correlation matrix. Secondly, by black-box modeling approximation, the input-output relation of the system through Kalman theory is made in order to make the system characteristic matrices optimal definite. Furthermore, by expressing Hankel matrix's multiplicities from Eigen solution of the system state matrix obtained in previous iteration, it is possible to determine both the covariance of nonmeasurable process noise and measurement noise matrixes which are present in the Riccati equation. This means that both measurement and process covariance noises' matrixes are indirectly built only from measured out-put data. The repetition of iterations is done until the error is sufficiently minimized. And then system modal parameters are extracted from these obtained system characteristic matrices. This system is used for modal update of the system in which modal parameters are applied directly and iterative methods. The code supporting this algorithm can be interfaced with the codes of the finite elements.

Keywords: Dynamic Parameters, Ambient Vibration, System Identification

ÖZET

Bu çalışmada, ortam titreşimi kullanılarak, modal titreşimde sistem karakteristik matrisinin geliştirilmesi ve düzeltilmesine dayanan yeni bir yaklaşım önerilmiştir. Sonuç, gerçek sistem karakteristik matrislerinin hatanın büyük ölüde asgariye indirilebileceği şekilde doğru bir şekilde oluşturulmasıdır. Bu, sistem parametrelerinin daha güvenilir bir şekilde güncellenebileceğini açıkça gösterir. Öncelikle, yaklaşık sistem karakteristik matrisleri, tepki korelasyon matrisinden inşa edilen blok Hankel matrisinin tekil değer ayrışımı kullanılarak belirlenir. İkinci olarak, kara kutu modelleme yaklaşımı ile sistem karakteristik matrislerini iyileştirmek için Kalman teorisiyle sistemin giriş-çıkış ilişkisi oluşturulur. Ayrıca, Hankel matrisinin çarpanlarını önceki iterasyonda elde edilen sistem durum matrisinin özdeğer çözümünden ifade ederek ölçülebilir olmayan işlem gürültüsünün kovaryansını ve Riccati denkleminde bulunan ölçüm gürültü matrislerini belirlemek mümkündür. Bu, ölçüm ve süreç kovaryans gürültülerinin matrislerinin yalnızca ölçülen çıktı verisinden dolaylı olarak oluşturulduğu anlamına gelir. Bu iterasyonlar, tahmini hatayı minimuma indirene kadar tekrarlanır. İterasyonların tekrarı, hata yeterince küçültülene kadar yapılır. Elde edilen sistem karakteristik matrislerinden sistem modal parametreleri çıkarılır. Bu sistem modal parametreleri, doğrudan ve iteratif yöntemlerin uygulandığı sistem modal güncellemesi için kullanılır. Bu algoritmayı destekleyen kod sonlu elemanlar kodlarıyla ara yüz haline getirilebilir.

Anahtar Kelimeler: Dinamik Parametreler, Çevresel Titreşim, Sistem Tanımlama

1. INTRODUCTION

In the condition where forced excitation tests are hard to be performed or when only response data can be measured without knowing actual loading conditions, the only technique to determine system identification of structure is to use operation modal analysis (output-only modal identification techniques). In this method, there is no need for an extra excitation to determine the dynamic parameters of the structure.(Tuhta, 2010; Ge and Lui, 2005; Wei, 1990; Raol and Madhuranath, 1996) and this is taken as its main advantage. In addition, in case with structures of high period and modes, it may be difficult to excite it with artificial shaking, whereas for drop weight or ambient sources, generally there is no problem. Despite this, ambient excitation cannot be used in case the mass-normalized mode shapes are needed. To develop reliable finite models of structures, Output-only modal identification technique efficiently used with model updating tools needs to be done. Output –

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Only Model Identification studies of systems and results performed in past years are appropriately shown in references of structural vibration solutions, partly in (Cunha et al. 2005). Concerning modal updating of the structure it is needed to estimate sensitivity of reaction of examined system to the change of parameters of a building as shown in (Kasimzade, Tuhta 2007, Kasimzade 2006). The definition of system identification as explained by researchers (Bendat, 1998; Ibrahim, 1977; Juang, 1994; Peeters, 2000; Roeck 2003; Balmes 1997; ARTeMIS, 2003; Ljung, 1999; Van Overschee and De Moor, 1996), it is a process that develops the mathematical expression of a physical system using experimental data. There are usually three types of definitions in engineering structures. These define modal parameter definition, structural-modal parameter setting and control model. (Andersen et al., 2007; Brownjohn and Carden, 2007; Gawronski, 2004; Johansson, 1993).In the frequency domain, the singular value decomposition of the spectral density matrix is e singular value frequency domain is denoted as frequency domain decomposition (FDD), which is further improved version enhanced frequency domain decomposition (EFDD). On the other hand, when the time domain is used, three different types of stochastic subspace identification (SSI) techniques are used. These species include the Unweighted Principal Component (UPC); Principal component (PC); And Canonical Variety Analysis (CVA). A new approach based on improvement and correction of system characteristic matrix in modal vibration from ambient vibration is shown below. In the first approximation of the algorithm, system characteristic matrices are determined by data-driven stochastic subspace identification method. In second approximation for estimating optimal state vector, applying steady-state Kalman filter to the stochastic steady space model equation makes the system characteristic matrices optimal definite (Labarre et al., 2003; Kalman, 1960; Kasimzade, 2006). Then all calculations are repeated until estimation error condition is satisfied.

2. DETERMINATION OF APPROXIMATE VALUES OF SYSTEM MATRICES

The purpose of the operational modal analysis is to determine modal parameters of a real system, which is assumed to be linear time invariant, by measuring only its response at specific locations, the exciting load is unknown. In the vibration analysis identification of actual system (black-box model) supported by experimental measurements, instead of determining the values of system characteristic matrices $[\hat{A}], [\hat{B}], [\hat{C}]$, appropriate unknown real matrices values appear first. The sequences for the analysis with the aim of this evaluation method are described below. The equations of motion of the continuous system are expressed as

$$[m]\{\ddot{u}(t)\} + [c]\{\dot{u}(t)\} + [k]\{u(t)\} = \{F(t)\} = [d]\{f(t)\}$$
(1)

These equations are then transformed to the state-space former of first order equations-i.e., a continuous-time state-space model of the system and are evaluated as shown in (Kowalczuk and Kozlowski, 2000).

$$\left\{\dot{z}(t)\right\} = \left[A_c\right]\left\{z(t)\right\} + \left[B_c\right]\left\{f(t)\right\}$$
(2a)

$$\begin{bmatrix} A_c \end{bmatrix} = \begin{bmatrix} [0] & [I] \\ -[m]^{-1}[k] & -[m]^{-1}[c] \end{bmatrix}$$
(2b)

$$\begin{bmatrix} B_c \end{bmatrix} = \begin{bmatrix} [0] \\ [m]^{-1} [d] \end{bmatrix}; \{z(t)\} = \begin{bmatrix} u(t) \\ \dot{u}(t) \end{bmatrix}$$
(2c)

With the condition that dynamic system is measured by the m_1 output quantities in the output vector $\{y(t)\}$ using sensors, for a system model expressed by equations (2), the appropriate measurement-output equation becomes

$$\{y(t)\} = [C_a]\{\ddot{u}\} + [C_v]\{\dot{u}\} + [C_d]\{u\}$$

= [C]{z(t)} + [D]{f(t)}
(3a)

$$[C] = [[C_a] - [C_a][m]^{-1}[k] \quad [C_v] - [C_a][m]^{-1}[c]]$$
(3b)

$$[D] = [C_a][m]^{-1}[d]$$
(3c)

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Where [m], [c], [k] mass, damping, stiffness matrices of the structure are constructed by finite element method (Kasimzade, 2005); $\{u\}$ is the displacement vector; $[A_c]$, is an $(n_1 = 2n_2)$; n_2 is the number of independent coordinates) by n_1 state matrix; [d] is an n_2 by r_1 input influence matrix, characterizing the locations and type of known inputs $\{f(t)\}; [C_a], [C_v], [C_d]$ are output influence matrices for acceleration, velocity, displacement respectively by using sensors; [C] is an $m_1 x n_1$ output influence matrix for the state vector $\{z\}$ and displacement only; [D] is an $m_1 x r_1$ direct transmission matrix; r_1 is the number of inputs; m_1 is the number of outputs. In the output-only modal analysis environment, the main assumption is that input force $\{F(t)\} = [d]\{f(t)\}$ comes from white noise or time impulse excitation. Under this hypothesis it may be possible to write a discrete-time stochastic state-space model as:

$$\{z_{k+1}\} = [A]\{z_k\} + [B]\{f_k\} + \{w_k\}$$
(4)

$$\{y_k\} = [C]\{z_k\} + [D]\{f_k\} + \{v_k\}$$
(5)

Where,

 $\{z_k\} = \{z(k\Delta t)\}$ is the discrete-time state vector;

 $\{w_k\}$ is the process noise due to disturbance and modeling imperfections;

 $\{v_k\}$ is the measurement noise due to sensors' inaccuracies;

 $\{w_k\}, \{v_k\}$ are non-measurable vectors, where they are assumed to be zero-averaged white noise. If this white noise acceptance is neglected; That is, if the input contains some dominant frequency components in addition to the white noise, these frequency components cannot be separated from the system's eigen frequencies and are determined as the poles of the system matrix. [A].

As shown from measurement-output equation (3) it indirectly depends on system model (2) and contains appropriate system mass, damping, rigidity matrices [m], [c], [k] respectively. For this reason, to carry out measurement by the relation (5), it will be required to know system model (2) with matrices [m], [c], [k] previously for **zero** approximation. For **zero** approximation these known matrices are denoted as [m], [c], [k] and they include $[A_c], [B_c], [A], [B], [C], [D]$ respectively.

In the real structures, exited by ambient vibration, the input $\{f(t)\}, \{f_k\}$ remains unmeasured and hence it disappears from the equation (2, 3, 4, 5) respectively. Then to take into consideration this fact, the input is implicitly modeled by the noise terms $\{w_k\}, \{v_k\}$ and mentioned relation becomes as:

$$\{z_{k+1}\} = [A]\{z_k\} + \{w_k\}$$
(6a)

$$\{\boldsymbol{y}_k\} = [C]\{\boldsymbol{z}_k\} + \{\boldsymbol{v}_k\}$$
(7a)

As said in above paragraphs, the main purpose of the operational modal analysis is the identification of modal parameters of the system from the output vector $\{y_k\}$, measured by the sensors located on the structure, whose dimensions are $n_{sensors} \times n_{samples}$. Where $n_{sensors}$ and $n_{samples}$ are the number of sensors and samples respectively. In the practical engineering of stochastic methods, the signal $\{y_k\}$ (it may be displacement, velocity or acceleration) given by sensors can be sampled at discrete time intervals depending on the characteristic of the computer processing.

Then by using the formulas (6a, 7a), measurements are realized and the system matrices $\begin{bmatrix} \hat{A} \end{bmatrix}$, $\begin{bmatrix} \hat{C} \end{bmatrix}$ are defined by the next sequences. Measuring the output vector $\{y_k\}$ and $\{y_s\}_{ref}$ (which is a subset of the output vector $\{y_{k+s}\}$) by the sensors located on the $s + k = 1 \div (s_* + k_*)$ characteristic and $s = 1 \div s_*$ reference points appropriately of the building structure, with the expected value operator denoted as E(...), the correlation function is calculated as:

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$$[R_k] = E(\{y_{k+s}\}\{y_s\}_{ref}^T) = \frac{1}{s_* - k} \sum_{s=0}^{s_* - k - 1} \{y_{k+s}\}\{y_s\}_{ref}^T$$
(8)

From which the following block Hankel matrix build as:

$$\begin{bmatrix} H_{p,q} \end{bmatrix} = \begin{bmatrix} [R_1] & [R_2] & \dots & [R_q] \\ [R_2] & [R_3] & \dots & [R_{q+1}] \\ \dots & \dots & \dots & \dots \\ [R_p] & [R_{p+1}] & \dots & [R_{p+q-1}] \end{bmatrix}$$
(9)

The factorization of the block Hankel data matrix is then realized by using singular value decomposition,

$$[H(0)] = [U][\Sigma][V]^{T}$$
⁽¹⁰⁾

Removing the Eigen values nearly zero from matrix $[\Sigma]$, consequently the defined matrices $[U_n], [\Sigma_n], [V_n]$ in accordance with its rank (n) are evaluated respectively.

The **first** approximation of the system matrices $\begin{bmatrix} \hat{A} \end{bmatrix}, \begin{bmatrix} \hat{B} \end{bmatrix}, \begin{bmatrix} \hat{C} \end{bmatrix}$ are calculated as:

$$\left[\hat{A}\right] = \left[\sum_{n}\right]^{-1/2} \left[U_{n}\right]^{T} \left[H(1)\right] \left[V_{n}\right] \left[\sum_{n}\right]^{-1/2}$$

$$\tag{11}$$

$$\left\lceil \hat{B} \right\rceil = \left[\sum_{n} \right]^{1/2} \left[V_{n} \right]^{T} \left[E_{r} \right]$$
(12)

$$\begin{bmatrix} \hat{C} \end{bmatrix} = \begin{bmatrix} E_m \end{bmatrix}^T \begin{bmatrix} U_n \end{bmatrix} \begin{bmatrix} \sum_n \end{bmatrix}^{1/2}$$
(13)

$$\begin{bmatrix} E_r \end{bmatrix}^T = \left(\begin{bmatrix} I_{n_1} \end{bmatrix} \begin{bmatrix} 0_{n_1} \end{bmatrix} \dots \begin{bmatrix} 0_{n_1} \end{bmatrix} \right), \begin{bmatrix} E_{m_1} \end{bmatrix}^T = \left(\begin{bmatrix} I_{m_1} \end{bmatrix} \begin{bmatrix} 0_{m_1} \end{bmatrix} \dots \begin{bmatrix} 0_{m_1} \end{bmatrix} \right)$$

Where [H(1)] is a shifted- block Hankel matrix; r_1 and m_1 are the number of inputs and outputs respectively; $[I_i]$ is an identity matrix of order *i*; $[0_i]$ is a null matrix of order *i*.

THE SYSTEM CHARACTERISTIC MATRICES' OPTIMAL DEFINITE 3.

In this section, it is required to reach theoretical target that is to find the best estimate $\{\hat{z}_k\}$ in the sense that the estimation error $\{e_k\} = \{z_k\} - \{\hat{z}_k\}$ is as small as possible, achievement to make definite **first** approximate values of the building system characteristics $\lceil \hat{A} \rceil, \lceil \hat{C} \rceil$ to the stochastic black-box models' is applied by Kalman filter theory. Theoretically, this type of filtering is very attractive, because it has a closed-form solution for its gain matrix. However, the Kalman filter requires information, including of covariance $E(\{w_k\}\{w_k\}^T) = [q\delta(k-j)] = [Q]$ of the non-measurable process noise $\{w(k)\}$ and covariance $E(\{v_k\}\{v_k\}^T) = [r\delta(k-j)] = [R]$ of the non-measurable measurement noise $\{v(k)\}$ (Nelson, 2000). Due to this reason, it is necessary to estimate these matrices ([Q], [R]) indirectly from measured output data $\{y_k\}$ and $\{y_s\}_{ref} = \{y_{k+s}\}$.

One of the crucial points (Kasimzade, 2006) in the presented method, is the determination of these matrices at least approximately $[Q] \cong [\overline{Q}], [R] \cong [\overline{R}] = [U_n]$ by expressing Hankel matrix's multiplicities from Eigen solution of the system state matrix $|\hat{A}|$ as shown below (In references (Juang, 1994) by Observer/Kalman Filter Identification method in order to avoid this problem. To make an effort to obtain estimate more approximately, Kalman filter gain is directly found from experimental data (Markov parameters) without estimating the covariance (P) of the process and measurement noises and solving Riccati equation).

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Briefly, in this procedure, make optimal definition stage of matrices $\begin{bmatrix} \hat{A} \end{bmatrix}, \begin{bmatrix} \hat{C} \end{bmatrix}$ for the **first** approximation, then the system model represented by the relation (6a, 7a) become as

$$\{z_{k+1}\} = \left[\hat{A}\right]\{z_k\} + \{w_k\}$$

$$\{y_k\} = \left[\hat{C}\right]\{z_k\} + \{v_k\}$$
(6b)
(7b)

By assuming and representing the system model as shown above, if the error covariance $[P] = E(\{e_k\}\{e_k\}^T)$ satisfies discrete algebraic Riccati equation, then a solution of this problem may be reached. The Sequences for solution of the formulated problem are given below.

Solving Eigen problem of the matrix $\begin{bmatrix} \hat{A} \end{bmatrix}$, consequently

$$\left(\left[\Lambda\right],\left[\Psi\right]\right) = eig\left(\left[\hat{A}\right]\right) \tag{14}$$

And expressing it in the form $[\hat{A}] = [\Psi][\Lambda][\Psi]^T$, then the Hankel matrix's multiplicities are express as:

$$[H_0] = \left[[U_n] [\Sigma_n]^{1/2} [\Psi] \right] \left[[\Psi]^{-1} [\Sigma_n]^{1/2} [V_n]^T \right] \approx \left[\overline{P} \right] \left[\overline{Q} \right]$$

$$\tag{15}$$

Where, $[\overline{R}] = [U_n], [\overline{P}] = [[U_n][\Sigma_n]^{1/2}[\Psi]], [\overline{Q}] = [[\Psi]^{-1}[\Sigma_n]^{1/2}[V_n]^T]$

Supporting this result is evaluation of Riccati equation:

$$[P] = \left[\hat{A}\right][P]\left[\hat{A}\right]^{T} - \left[\hat{A}\right][P]\left[\hat{C}\right]^{T}\left[\left[\bar{R}\right] + \left[\hat{C}\right][P]\left[\hat{C}\right]^{T}\right]^{-1}\left[\hat{C}\right][P]\left[\hat{A}\right]^{T} + \left[\bar{Q}\right]$$

$$\tag{16}$$

There are some conditions for the solution of the Riccati equation. The solution is only possible if the correlation function is positive. Therefore, there are several suggestions in the literature to guarantee the solution. If a solution of the Riccati equation exists, after defining it as $[P] = \lceil \hat{P} \rceil$, one can obtain Kalman gain of the building structure model:

$$\begin{bmatrix} \hat{K} \end{bmatrix} = \begin{bmatrix} \hat{A} \end{bmatrix} \begin{bmatrix} \hat{P} \end{bmatrix} \begin{bmatrix} \hat{C} \end{bmatrix}^T \begin{bmatrix} [\overline{R}] + \begin{bmatrix} \hat{C} \end{bmatrix} \begin{bmatrix} \hat{P} \end{bmatrix} \begin{bmatrix} \hat{C} \end{bmatrix}^T \end{bmatrix}^{-1}$$
(17)

Kalman filter equation is then evaluated as

$$\left\{\hat{z}_{k+1}\right\} = \left[\left[\hat{A}\right] - \left[\hat{K}\right]\left[\hat{C}\right]\right]\left\{\hat{z}_{k}\right\} + \left[\left[\hat{B}\right] - \left[\hat{K}\right]\left[D\right]\right]\left\{f_{k}\right\} + \left[\hat{K}\right]\left\{y_{k}\right\}$$

$$\tag{18}$$

With the output measurement $\{y_k\}$ satisfying

$$\{y_k\} = \left[\hat{C}\right]\{\hat{z}_k\} + \{\varepsilon_k\}$$
⁽¹⁹⁾

The output residual $\{\varepsilon_k\}$ satisfies $\{\varepsilon_k\} = [\hat{C}]\{e_k\} + \{v_k\}$.

Making a comparison of building system modeling by the Kalman filter equations (18, 19) with the first step system modeling equations,

$$\{z_{k+1}\} = \begin{bmatrix} \hat{A} \end{bmatrix} \{z_k\} + \{w_k\}; \{y_k\} = \begin{bmatrix} \hat{C} \end{bmatrix} \{z_k\} + \{v_k\}$$

The error is evaluated as: -the state estimation error

$$\{e_k\} = \{z_k\} - \{\hat{z}_k\}; \ \{E(e_k)\} = \{0\}$$
(20)

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-error dynamics

$$\{e_{k+1}\} = \left[\left[\hat{A}\right] - \left[\hat{K}\right]\left[\hat{C}\right]\right]\{e_k\} - \left[\hat{K}\right]\{v_k\} + \{w_k\}$$

$$(21)$$

-output residual

$$\{\varepsilon_k\} = [\hat{C}]\{e_k\} + \{v_k\}; \ \{E(\varepsilon_k)\} = \{0\}$$

$$(22)$$

If the condition (22) is not satisfied, all calculations are repeated until satisfaction is obtained. For example, if the condition (22) is not satisfied, in **second** approximation the case relation (6a, 7a) becomes

$$\{z_{k+1}\} = \begin{bmatrix} \hat{A} \end{bmatrix} \{z_k\} + \{w_k\}; \quad \{y_k\} = \begin{bmatrix} \hat{C} \end{bmatrix} \{z_k\} + \{v_k\}$$
(23)

Measurements in **second** approximation case are realized by these formulas and then system characteristic matrices $\left[\hat{A}\right], \left[\hat{C}\right]$, are defined by the operations (8-13).

To make an optimal definition of the system's matrices $\lceil \hat{A} \rceil, \lceil \hat{C} \rceil$, the relation in (6b, 7b) becomes as

$$\{z_{k+1}\} = \left[\hat{A}\right]\{z_k\} + \{w_k\}; \ \{y_k\} = \left[\hat{C}\right]\{z_k\} + \{v_k\}$$
(24)

For this-**second** approximation case, in all of the relations (14-22), matrices $\begin{bmatrix} \hat{A} \end{bmatrix}$, $\begin{bmatrix} \hat{C} \end{bmatrix}$ will take place instead of matrices $\begin{bmatrix} \hat{A} \end{bmatrix}$, $\begin{bmatrix} \hat{C} \end{bmatrix}$, respectively. And a result from these iterations (when condition (22)satisfied) are definite real building system parameters, in another words, system is identified completely and obtained system matrices will be marked as $\begin{bmatrix} \overline{A} \end{bmatrix}$, $\begin{bmatrix} \overline{C} \end{bmatrix}$ for the illustrating of operations in next section (Tuhta, 2010; Phan and Longman, 2004).

Obtained modal parameters-damping, period (which contain the eigen value matrix), mode shape used as a reference modal "experimental" data (meaning that they are defined to support experimental result) the analytical (finite element model) (Chen, 2001) stiffness, damping and mass matrices are corrected by the known direct (Caesar, 1986) and iterative updating methods (Link, 1993) using convergence criteria (Dascotte and Vanhonacker, 1989). to speed up updating procedure, the marked updating parameters previously are fuzzyfied in the user definable intervals of these parameters using full factorial and orthogonal array testing on the base finite element method (Kasimzade, 2002).

4. NUMERICAL EXAMPLE

Three DOF systems with mass (m), classical damping (c) and stiffness (k) under unit step excitation (Fig.1) applied at DOF 3 and velocity measurement from third mass is recorded. In the case (1) the system is examined without noise and some modal result are represented in Table 1. In the case (2) the output data is polluted with Gaussian zero-mean white noise 1% of the unpolluted time histories and result are presented in the same table. In the case (3) after improving system characteristic matrices (for the case 2) by the above-mentioned algorithm obtained by modal parameters presented in the same table. It can be seen in Table 1 that the identified samples are very close to the exact values. System periods are acceptable, and as seen damping ratios are more sensitive to the noise.

$$m = \begin{bmatrix} 181.37 & 0 & 0 \\ 0 & 181.37 & 0 \\ 0 & 0 & 181.37 \end{bmatrix} kN \sec^2 / m$$
$$c = \begin{bmatrix} 615.48 & -224.33 & 0 \\ -224.33 & 615.48 & -224.33 \\ 0 & -224.33 & 391.15 \end{bmatrix} kN \sec/m$$

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(b)





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Figure 2. (a) Input Excitation-Time, (b) Response (y) Without Noise-Time, (c) Response (y) Without Noise-Time Graphics and (d) Hankel Singular Values

Mode No	Case (1)		No Case (1) Case (2)		Case (3)	
	Т	ųς	Т	ξ	Т	ξ
1	0.58	0.05	0.57	0.05	0.59	0.05
2	0.21	0.06	0.20	0.05	0.22	0.06
3	0.14	0.05	0.13	0.05	0.15	0.05

Table 1. Identified Samples Results

5. CONCLUSION

The correction of the system characteristic matrices in modal identification through ambient vibration is presented. In this algorithm, in the first approximation, determined are the actual system characteristic matrices by the data-driven stochastic subspace identification method. In second approximation, an optimal estimation state vector, that is to make the system characteristic matrices optimal definite, is obtained by applying the steady-state Kalman filter to the stochastic state-space model equation. All calculations are repeated until the estimation error condition is satisfied (Table 1).

Another word process and measurement noises covariance matrixes indirectly are constructed only from measured output data. These iterations are repeated until satisfying estimated error. As a result of these iterations, actual system characteristic matrices are determined more accurately with minimum error. Then from the determined system, characteristic matrices are extracted system modal parameters (case 3). As a result of this approach, it has been shown that the actual system characteristic matrices are more accurately determined by the minimum error. These values are considered to be more reliable in updating the system parameters.

To speed up updating process, marked updating parameters are previously fuzzified in the user definable intervals of these parameters by using full factorial and orthogonal array testing on the base finite element method. And finally, a prepared code by finite element method can be interfaced in order to support the above algorithm (Appendices). Similar results with the generated algorithm are obtained and presented for multi degree of freedom systems (4 DOF steel structures and 11 DOF concrete structures), (Tuhta, 2010).

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7. APPENDICES					
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Algorithm of Optimal Determination of S	tructural Dynamical Parameter	s Using Ambient Vibration			
	Measurement of Structure Responses	s oblig random violation			
	Creation of Correlation Matrix				
	Ų				
Creation o	f Zero and First Order Block Hankel Matrice	s			
	₹ 8				
Separation of the Hankel Matrix (SVD)					
Separation of the Hankel Matrix (SVD) ↓ The Elimination of Those Close to Zero of the Eigenvalue Matrix and the Establishment of Suitable Matrix Components of This Rank ↓ Calculation of Observational Markov Parameters (Kalman Gain) ↓ Calculation of System and Observational Gain Markov Parameters					
The Elimination of Those Close to Zero of the Eigenvalue Matrix and the Establishment of Suitable Matrix Components of This Rank					
	ation of Observational Markov Parameters				
	(Kalman Gain)				
Calculation of S	System and Observational Gain Markov Parar	neters			
Creation of System Matrice	s Using the Above Results and the First Degr	ee Hankel Matrix			
	Ų				
Observability, Controllability Matrices, Unmeasured Process Noise C	•	Iatrices by Solving the Eigenproblem of the System Matrix			
	↓				
Calculation of	Kalman Gain by Dissolving the Riccati Equa	ation			

↓ Creation of a Kalman Filter ↓

Evaluation of System Output Residual (ε_{ϵ}) \downarrow $E(\varepsilon_{\epsilon}) \cong 0$ \downarrow $\langle \underline{[YES]} \rangle$ \downarrow Modal Parameters (Angular Frequencies (ω) , Damping Ratios (ξ) , Period s (T))

 $\langle \overline{NO} \rangle \blacktriangleleft$