European Journal of
Engineering and Applied Sciences

ISSN: 2651-3412 (Print) & 2667-8454 (Online)
Journal homepage: http://dergipark.gov.tr/EJEAS
Published by Çorlu Faculty of Engineering, Tekirdağ Namık Kemal University

European J. Eng. App. Sci 1(1), 9-13, 2018

Research Article

TiO_2 Memristor with Logarithmic Memristance

Reşat Mutlu[1,*]

[1] **Department of Electronics and Telecommunication Engineering, Çorlu Faculty of Engineering, Tekirdağ Namık Kemal University, Tekirdağ, Turkey**

Received: 13.07.2018 *Accepted: 09.11.2018*

Abstract: An ideal memristor, which was claimed to be the fourth fundamental element of circuit design by Dr. Chua in 1971, is a nonlinear resistor and its properties cannot be mimicked with linear time-invariant circuit elements. A thin-film which behaves as if a memristor has been declared found experimentally by a HP research team lead by Stanley Williams in 2008. A quite explicit model of the memristor has also been given by the team. The HP memristor resistance can be found by summing the resistances of the doped and undoped regions. Assuming the doped region length is proportional to memristor charge, which is the integration of memristor current, the doped region has a constant drift speed, and a constant memristor cross-section, the HP memristor resistance has a linear charge dependency till it saturates. In this paper, it is shown that a memristor with a logarithmic charge dependency can be made using the principles given by the team and making some modifications to memristor geometry.

Keywords: *TiO_2 memristor model, Memristance calculation, Programmable gain amplifier, Manufacturing tolerances, Circuit topology.*

Logaritmik Memristanslı TiO_2 Memristör

Özet: İdeal bir memristör, 1971 yılında Dr. Chua tarafından devre tasarımı için dördüncü temel devre elemanı olarak iddia edilen nonlinear bir dirençtir ve özellikleri lineer zamanla-değişmeyen devre elemanları tarafından taklit edilememektedir. Memristör olarak davranan bir ince-filmin bulunduğu, 2008 senesinde Stanley Williams tarafından yönetilen bir HP araştırma timi tarafından ilan edilmiştir. Bu memristörün oldukça anlaşılabilir bir modeli de bu tim tarafından verilmiştir. Bu HP memristör direnci katkılanmış ve katkılanmamış bölgelerin dirençlerini ekleyerek bulunabilir. Katkılanmış bölge uzunluğunun akımın integrali olan memristör yüküne orantılı olduğu, katkılanmış bölgenin sabit bir sürüklenme hızına sahip olduğu ve memristör kesiti sabit olarak kabul edilirek, HP memristör direncinin, doyma gerçekleşene kadar, lineer yük bağımlılığı vardır. Bu makalede, logaritmik yük bağımlılığı olan bir memristörün HP timi tarafından verilen prensipleri kullanarak ve memristör geometrisine bazı değişiklikler yapılarak yapılabileceği gösterilmiştir.

Anahtar Kelimeler: *TiO_2 memristör modeli, Memristans hesaplama, Programlanabilir güç kuvvetlendiricisi, Üretim toleransları, Devre topolojisi.*

Geliş: 13.07.2018 *Kabul: 09.11.2018*

[*] Corresponding author.
E-mail address: rmutlu@nku.edu.tr (R. Mutlu)

1. Introduction

A memristor is a new-found circuit element, which has been theoretically predicted as the fourth fundamental circuit element by Chua in 1971 [1]. Memristor is a dissipative circuit element like resistor but it has a relationship between its flux-linkage, which is its voltage integration, and its charge, which is its current integration. Its voltage to current ratio depends on the charge, which past through it, and is called memristance. For a long time, memristor has been regarded as a mathematical work or a mathematical curiosity since no memristor was found. In 2008, a HP research team led by Stanley Williams has declared that they have found a memristor in nanoscale made of TiO_2 sandwiched between Pt contacts [2]. More and more papers about memristor have started appearing in literature and a review paper on memristor can be found in [3]. Until now, the most explicit memristor model has been given by the HP team [2]. It is a first order memristance model, i. e., it has a linear charge dependency and ignores nonlinear dopant drift or current dependency [3-5]. The most commonly-used memristor model in literature is also called memristor model with linear dopant drift. In this paper, it is shown that memristor with a logarithmic charge dependency can be obtained using the principles given for the first order memristor by Williams' team in [2].

The paper is arranged as following. In the second section, the memristor model with linear dopant drift is explained. In the third section, it is theoretically shown how to make a memristor with a logarithmic charge dependency. In the fourth section, a possible application for it is predicted. The results are summarized in Conclusion section.

2. Memristor Model with Linear Dopant Drift Speed

The memristor, William's team has found, is actually complex but a first order memristor is what they have presented in [2] and it is so easy to analyze. When a positive voltage is applied as shown in Figure 1, Oxygen ions starts diffusing within TiO_2. If TiO_2 is fully doped with Oxygen ions, its memristance becomes minimum and equal to R_{ON}. If TiO_2 is not doped at all with Oxygen ions, its memristance becomes maximum and equal to R_{OFF}. According to their model, the diffused charge is proportional to the diffusion length. If the doped region has a length of w, the memristance, M(q), becomes equal to the total resistance of the doped and undoped regions;

$$\mathrm{M(q)} = R_{ON}\, w/D + R_{OFF}\,(D-w)/D \qquad (1)$$

Considering saturation, at w=D or at q=q_{sat},

$$\mathrm{M}(\mathrm{q}_{sat}) = R_{ON} \qquad (2)$$

Where

q_{sat} is the maximum doped charge or the maximum memristor charge.

At q=0 or at w=0, i.e., there is no doped region,

$$\mathrm{M}(0) = R_{OFF} \qquad (3)$$

Therefore, the memristance is linearly dependent on memristor charge [2]. In [2], memristance has also been given of the following form;

$$\mathrm{M(q)} = \mathrm{M}_0 - Kq \qquad (5)$$

Where

$\mathrm{M}_0 = R_{OFF}$ is the resistance if the memristor region were fully undoped.

K is the charge coefficient of the memristor.

3. TiO_2 Memristor with Logarithmic Memristance

If the TiO_2 memristor is modified to have a radial geometry, which is defined as in Figure 2 with α being the angle of platinum contacts and TiO_2 being disposed between the radiuses R_1 and R_2, using the physical parameters, α, R_1 and R_2, and the electrical conductivity σ, its memristance can be calculated. If TiO_2 is fully doped, let its resistance equal to R_{on} and if not doped at all let its resistance be equal to R_{OFF} or M_o. For such geometry, If Memristor is not doped at all with O_2 ions, the resistance of the undoped TiO_2, is equal to

$$\mathrm{R}_{OFF} = \ln(R_2/R_1)/(\alpha\sigma_2 l) \qquad (6)$$

Where σ_2 is the electrical conductivity of the undoped TiO_2.

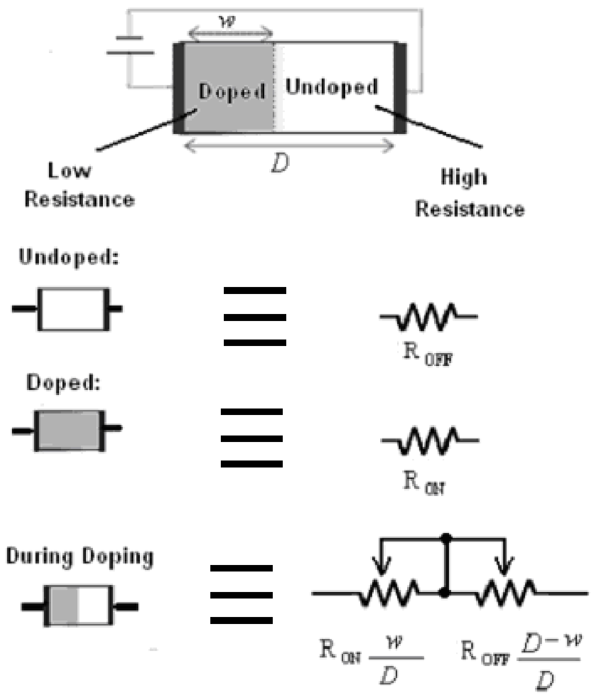

Figure 1. TiO_2 memristor and its equivalent circuit

If Memristor is fully doped with oxygen ions, the resistance of the fully doped TiO_2, is equal to

$$\mathrm{R}_{ON} = \ln(R_2/R_1)/(\alpha\sigma_1 l) \qquad (7)$$

Where σ_1 is the electrical conductivity of the fully doped TiO_2.

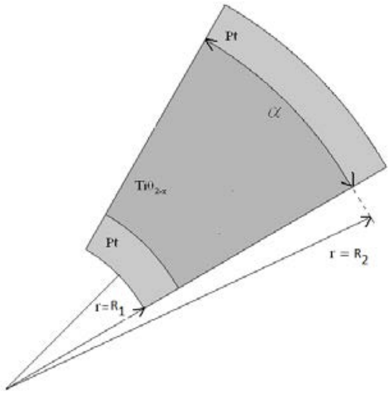

Figure 2. The unsaturated cylindrical-cut memristor topology

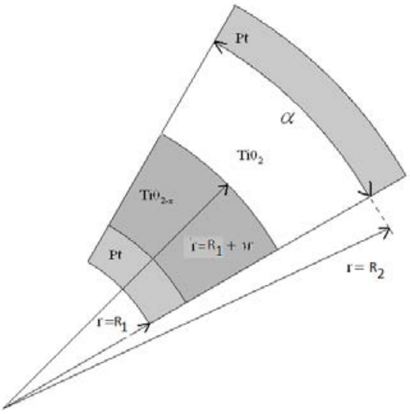

Figure 3. The cylindrical-cut memristor topology during doping

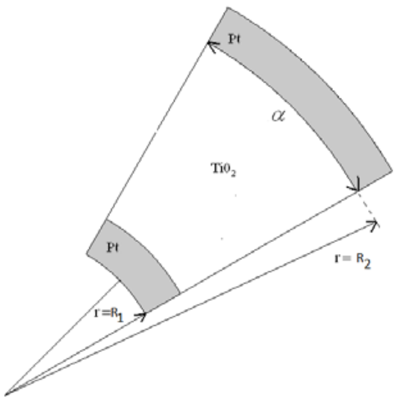

Figure 4. The saturated cylindrical-cut memristor topology

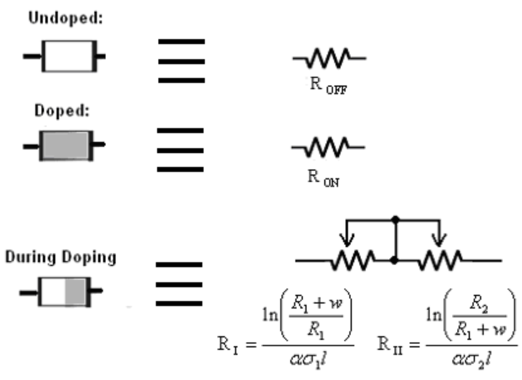

Figure 5. The cylindrical-cut memristor equivalent circuit

If Memristor is doped by oxygen ions by the radius, $R_1 + w$, the resistance of the doped region is

$$\mathrm{R}_{\mathrm{I}} = \ln\big((R_1 + w)/R_1\big)/(\alpha\sigma_1 l) \tag{8}$$

and the resistance of the undoped region is

$$\mathrm{R}_{\mathrm{II}} = \ln\big(R_2/(R_1 + w)\big)/(\alpha\sigma_2 l) \tag{9}$$

The total memristance or the total resistance is

$$\mathrm{M(q)} = \mathrm{R}_{\mathrm{I}} + \mathrm{R}_{\mathrm{II}} \tag{10}$$

$$\mathrm{M(q)} = \frac{1}{\alpha l}\left(\frac{\ln\big((R_1+w)/R_1\big)}{\sigma_1} + \frac{\ln\big(R_2/(R_1+w)\big)}{\sigma_2}\right) \tag{11}$$

Using these equations and assuming that the charge diffused area is proportional to memristor charge, the diffusion radial thickness can be calculated as the following.

$$\mathrm{w} = \sqrt{\mathrm{R}_1^2 + (\mathrm{R}_2^2 - \mathrm{R}_1^2)(q/q_{sat})} - \mathrm{R}_1 \tag{12}$$

Therefore, the memristance as a function of memristor charge can be written as

$$\mathrm{M(q)} = \ln\Big(R_2^{\sigma_2} R_1^{-\sigma_1}\big((q/q_{sat})(R_2^2 - R_1^2) + R_1^2\big)^{\sigma_1-\sigma_2}/(\alpha l)\Big) \tag{13}$$

or

$$\mathrm{M(q)} = \frac{\sigma_1-\sigma_2}{\alpha l}\ln\left(\frac{R_2^{\sigma_2}}{R_1^{\sigma_1}}\left(\left(\frac{q}{q_{sat}}\right)\left(\frac{R_2^2}{R_1^2}-1\right)+1\right)\right) + \frac{\sigma_1-\sigma_2}{\alpha l}\ln\left(R_1^2\right) \tag{14}$$

If

$$\mathrm{M}_0 = (\sigma_1 - \sigma_2)\ln\left(R_1^2\right)/\alpha l \tag{15}$$

$$A = (\sigma_1 - \sigma_2)/\alpha l \tag{16}$$

and

$$B = R_2^{\sigma_2} R_1^{-\sigma_1}\left((R_2/R_1)^2 - 1\right)/q_{sat} \tag{17}$$

If the memristor is not saturated, its memristance can be written as the following;

$$\mathrm{M(q)} = M_0 + A.\ln(Bq + 1) \tag{18}$$

Where

M_o is the undoped resistance.

A and B are charge coefficients of the memristance.

(14) or (18) shows that a memristor with a logarithmic charge dependency can be obtained using Pt-TiO_2-Pt materials with the cylindrical-cut topology given in Figure 2.

4. A Possible Application for Memristor with Logarithmic Memristance

There may be some possible application areas for the logarithmic memristor. Memristors are considered for being used in programmable gain amplifiers [6-8]. A M-R programmable inverting amplifier is shown in Figure 6. A logarithmic memristor has a different characteristic than a linear charge dependent memristor. If the memristor is not saturated, the charge sensitivity of a logarithmic memristor and that of a linear charge dependent memristor are respectively equal to

$$\frac{dM(q)}{dq} = \frac{\text{b.c}}{\text{c.}q+1} \quad \text{and} \quad \frac{dM(q)}{dq} = -K \tag{19}$$

If the needed potentiometer range is wide, instead of a linear charge dependent memristor, a logarithmic charge dependent memristor may be used in a programmable gain amplifier.

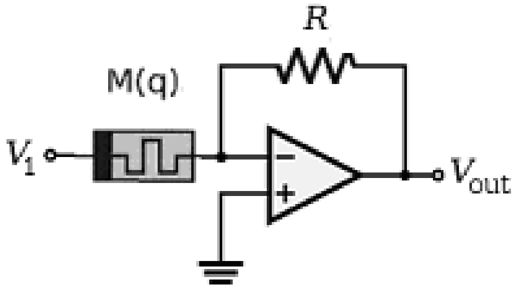

Figure 6. M-R programmable gain amplifier [6]

5. Conclusion

It is shown that a memristor with a logarithmic memristance can be easily obtained by playing with the structure of the TiO_2 memristor given in [2]. We would like to verify logarithmic memristance experimentally but we couldn't do it due to the fact that we don't have the necessary infrastructure and tools for it. If we were able to do the experiments, we can expect that the logarithmic charge dependence may exist only in some operation area of the device since we have ignored the nonlinear dopant drift when deriving formulas. When the nonlinear dopant drift memristor model is developed more, we can understand how much logarithmic memristance we can also obtain.

The effect of manufacturing tolerances on memristor performance has also been under study in literature [9-12]. We think that the effect of the logarithmic charge dependence or nonlinear charge dependence may also appear in the TiO_2 memristors with big manufacturing tolerances. Therefore, future manufacturers or researchers must be wary of the possibility that some tolerances on platinum contacts or contact cross-section of TiO_2 memristor may result in a logarithmic memristance characteristic and its effect on memristor parameters, memristor power, switching time etc must be studied.

It may also be worth mentioning that Hodgkin–Huxley model, which was also an inspiration motivating Chua to predict the possible existence of the new circuit component memristor, has nonlinear memristances and those must also be dependent on varying channel thicknesses. We believe that emulator circuits of Hodgkin–Huxley model may be approximately made using logarithmic memristors (by playing memristor structure) and can be used for teaching biomedical students in circuit labs. We have also predicted that a possible application for the logarithmic memristor may be in programmable gain amplifiers when there is a need for a wider operation range.

When, in future, there is more information on memristors, new kind of memristance functions of memristors or memristive systems can be obtained by playing or shaping its structure. There must be limits for the radial angle, α , due to manufacturing methods, it may not allow us to obtain every logarithmic function, and this must be also inspected experimentally. There is still so much to discover about the new circuit element memristor and its potential only can be realized after its inspection using new topologies and new materials.

References

[1] Chua, L. (1971). Memristor-the missing circuit element. *IEEE Transactions on circuit theory*, 18(5), 507-519.

[2] Strukov, D. B., Snider, G. S., Stewart, D. R., & Williams, R. S. (2008). The missing memristor found. *Nature*, 453(7191), 80.

[3] Kavehei, O., Iqbal, A., Kim, Y. S., Eshraghian, K., Al-Sarawi, S. F., & Abbott, D. (2010, August). The fourth element: characteristics, modelling and electromagnetic theory of the memristor. In *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences* (Vol. 466, No. 2120, pp. 2175-2202). The Royal Society.

[4] Joglekar, Y. N., & Wolf, S. J. (2009). The elusive memristor: properties of basic electrical circuits. *European Journal of Physics*, 30(4), 661.

[5] McDonald, N. R., Pino, R. E., Rozwood, P. J., & Wysocki, B. T. (2010). Analysis of dynamic linear and non-linear memristor device models for emerging neuromorphic computing hardware design. In *IEEE World Congress on Computational Intelligence 2010.*

[6] Pershin, Y. V., & Di Ventra, M. (2010). Practical approach to programmable analog circuits with memristors. *IEEE Transactions on Circuits and Systems I: Regular Papers*, 57(8), 1857-1864.

[7] Shin, S., Kim, K., & Kang, S. M. (2011). Memristor applications for programmable analog ICs. *IEEE Transactions on Nanotechnology*, 10(2), 266-274.

[8] Wey, T. A., & Jemison, W. D. (2011). Variable gain amplifier circuit using titanium dioxide memristors. *IET circuits, devices & systems*, 5(1), 59-65.

[9] Chen, Y., & Wang, X. (2009). Compact modeling and corner analysis of spintronic memristor. In *Proceedings of the 2009 IEEE/ACM International Symposium on Nanoscale Architectures* (pp. 7-12).

[10] Jo, K. H., Jung, C. M., Min, K. S., & Kang, S. M. (2010).

Self-adaptive write circuit for low-power and variation-tolerant memristors. *IEEE Transactions on Nanotechnology*, 9(6), 675-678.

[11] Hu, M., Li, H., Chen, Y., Wang, X., & Pino, R. E. (2011, January). Geometry variations analysis of TiO 2 thin-film and spintronic memristors. In *Design Automation Conference (ASP-DAC), 2011 16th Asia and South Pacific* (pp. 25-30). IEEE.

[12] Rajendran, J., Karri, R., & Rose, G. S. (2011, May). Parallel memristors: Improving variation tolerance in memristive digital circuits. In *Circuits and Systems (ISCAS), 2011 IEEE International Symposium* on (pp. 2241-2244). IEEE